淡江大學 97 學年度轉學生招生考試試題

1C-1

系別：數學學系二年級　　　　科目：線 性 代 數

可否使用計算機			
可		否	✓

本試題共 1 大題， 一 頁

Partial credit—You must show all your work.

1. (10 %) Consider the system of equations

$$\begin{aligned} 2x_1 + 5x_2 - x_3 &= a \\ x_1 + 2x_2 \quad\quad &= b \\ 3x_1 + 7x_2 - x_3 &= c \end{aligned}$$

Determine conditions on a, b, c that are necessary and sufficient for the system to be consistent.

2. (10 %) Let S_1 and S_2 be subspaces of a vector space V. Suppose $S_1 \neq \{0\}$, $S_1 \neq V$, $S_2 \neq \{0\}$ and $S_2 \neq V$. Show that there exists a vector $\mathbf{v} \in V$ such that $\mathbf{v} \notin S_1$ and $\mathbf{v} \notin S_2$.

3. (10 %) Let $A = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 2 & 1 \\ 3 & 5 & 3 \end{bmatrix}$. Find A^{-1}.

4. Let $M_{33} = \left\{ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \middle| a_{ij} \in \mathbb{R} \right\}$ be the vector space of all 3×3 matrices. Define a linear transformation $T : M_{33} \to \mathbb{R}$ by

$$T\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right) = a_{11} + a_{22} + a_{33}.$$

(1) (10 %) Find a basis for $\ker T$.

(2) (10 %) Find $\operatorname{nullity} T$ and $\operatorname{rank} T$.

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by

$$T(a, b, c) = (2a + b, b - c, c - 3a).$$

Let $\mathcal{B}_1 = \{(1, 1, 0),\ (1, 0, 1),\ (0, 1, 0)\}$ and $\mathcal{B}_2 = \{(1, 0, 0),\ (0, 1, 0),\ (0, 0, 1)\}$ be two bases of $\mathbb{R}^3$.

(1) (10 %) Find $[T]_{\mathcal{B}_1}$ (the matrix of T with respect to the basis $\mathcal{B}_1$) and $[T]_{\mathcal{B}_2}$ (the matrix of T with respect to the basis $\mathcal{B}_2$).

(2) (10 %) Find a matrix P such that $P^{-1}[T]_{\mathcal{B}_1}P = [T]_{\mathcal{B}_2}$.

6. (10 %) Let A be a 3×3 real orthogonal matrix. Suppose $\lambda \in \mathbb{R}$ is an eigenvalue of A. Show that either $\lambda = 1$ or $\lambda = -1$.

7. (10 %) Let $T : V \to W$ be a one-to -one linear transformation with $\dim V = n = \dim W$. Show that T is onto.

8. (10 %) Let $A = \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$. Find an invertible matrix P that makes $P^{-1}AP$ a diagonal matrix.