淡江大學96學年度轉學生招生考試試題

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系別：數學學系二年級　　　　科目：線 性 代 數

可否使用計算機			
可		否	✓

本試題共 1 頁

1. Find a matrix A that satisfies the following equation
   $$(A^T\begin{pmatrix}4 & 0\\ 1 & -1\end{pmatrix})^{-1}=\begin{pmatrix}1 & -2\\ -1 & 3\end{pmatrix}$$ (10%)

2. let $T: R^3 \to R^3$ be defined by
   $T(x, y, z)=(2x+y, y-3z, 2y+8z)$
   a) Find a matrix representation of T with respect to the standard bases. (5%)
   b) Find the characteristic polynomial, eigenvalues and eigenvectors of T. (10 %)
   c) Is T diagonalizable ? Give your reason. (5%)

3. Let $P_2(R)$ be the set of polynomial with degree $\leq$ 2 and $(P_2(R), <\cdot,\cdot>)$ be an inner product space with $<p(x), q(x)>=\int_0^1 p(x)q(x)dx$ for any $p,q \in P_2(R)$. Let $W=\{p\in P_2(R), p(0)=0, p(1)=0\}$
   a) Show that W is a linear subspace of $P_2(R)$ (5%)
   b) Find an orthonormal basis of W. (5%)
   c) Find an orthonoraml basis of $W^{\perp}$, where $W^{\perp}$ is the orthogonal complement of W in $P_2(R)$. Give your reason. (10 %)

4. Let <. , .> be an inner product on a finite dimensional vector space V. Let $T: V\to V$ be an isomorphism. For any v, w in V, define <<v,w>>=<T(v),T(w)>, show that << . , .>> is an inner product on V. (10%)

5. Let V be a vector space and $T: V\to V$ be a linear transformation satisfying $T^2=T$
   a) Show that $V=KerT\oplus \operatorname{Im}T$ (10 %)
   b) Let $P_2(R)$ be the set of polynomial with degree $\leq$ 2. Let $T: P_2(R)\to P_2(R)$ be defined by $T(a+bx+cx^2)=(a-b+c)(1+x+x^2)$. Show that $T^2=T$ and find a matrix representation of T. (10 %)

6. Let U, W be subspaces of a vector space V. Suppose that $U\neq O$, $W\neq O$ and $U\cap W=\{O\}$, here O is the zero element of V. Show that if $u\in U$, $w\in W$, then u , w are linearly independent. (10 %)

7. Let A, B be n by n matrices. Suppose that the system (AB) x = 0 has only zero vector solution, show that Ax = 0 has only zero vector solution. (10 %)