淡江大學 95 學年度轉學生招生考試試題

系別：數學學系二年級　　　　科目：線 性 代 數　　　　11-1

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁

1. Let $A=\begin{bmatrix} 6 & -5 \\ 2 & -1 \end{bmatrix}$.

   (a) Find a matrix P that diagonalizes A. (10 points)

   (b) Find A^{10}. (10 points)

2. Let V be the space consisting of all polynomials of degree less than or equal 2 and the zero polynomial. Let T : V→ R^2 be defined by T(a+bx+cx^2) = (a+b, c) and B={1, x, x^2}, D={(1,-1),(1,1)}.

   (a) Find the ker(T). (10 points)

   (b) Find the matrix of T corresponding to the ordered bases B and D. (10 points)

3. Let A=$\begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & -2 & 2 & 4 \\ 1 & -1 & 0 & 3 \end{bmatrix}$ be 3×4 matrix. (20 points)

   (a) Show that AX=Y is consistent for all 3×1 matrix Y.

   (b) Find a basis for the solution space of AX=0.

4. Let u_1= (1,1) and u_2= (1, -1), and let T : $R^2 \to R^2$ be the linear operator such that

   T(u_1)= (1, -2) and T(u_2)=(-4, 1)

   Find a formula for T(x, y). (10 points)

5. Let W be a finite-dimensional subspace of an inner product space V. Let {v_1, v_2, …v_n} be an orthonormal basis for W. Let x∈ V and

   $$y=\langle x, v_1\rangle v_1+\langle x, v_2\rangle v_2+\ldots+\langle x, v_n\rangle v_n.$$

   Prove that

   (a) x -y is orthogonal to W. (10 points)

   (b) ||x-y||< ||x-z|| for all z in W that is different from y. (10 points)

6. Let A be a n×n real matrix. Prove that if A^2+ I= 0, where I is the identity matrix, then n is even. (10 points)