淡江大學九十四學年度轉學生招生考試試題

系別： 數學系二年級　　　　　　科目：線　性　代　數

准帶項目請打「V」	
X	簡單型計算機

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1. Let $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 3 \end{bmatrix}$。

(a) Find the characteristic polynomial of A. (5 points)
(b) Find eigenvalues and eigenvectors of A. (5 points)
(c) Find the minimal polynomial of A. (5 points)
(d) Show that A is not diagonalizable. (5 points)
(e) Let f(t)=(t-2)(t+1)t(t-2) - 2t – 3. Find f(A). (5 points)
(f) Show that A is invertible and find its inverse A^{-1}. (5 points)

2. Show that if A is $n \times n$, with A is skew symmetric($A= -A^T$), and n odd, then det(A) = 0. (10 points)

3. Let A and B be matrices of sizes $m \times n$ and $n \times p$ respectively. Show that rank (AB) $\leq$ rank (A). (10 points)

4. Find a polynomial P(x) of degree 2 such that P(0)= 2, P(-1)=3, P(2)=4. (10 points)

5. Suppose that {x, y} is a linearly independent set in a vector space V. Show that if $T : V \to W$ is a one –to-one linear transformation, then {T(x+y), T(2x-3y)} is also linearly independent. (10 points)

6. Show that {(1,-1,0), (1,1,0), (1,1,1)} is a basis in R^3.(10points)

7. Let V be the space consisting of all polynomials of degree less than or equal 2 and the zero polynomial. Let $B=\{x^2, x, 1\}$ Let T be the linear transformation from V to V defined by T(p(x))=p(x-1). Find the matrix of T with respect to the basis B. (10points)

8. Let U = span{ s, t} in R^4, where $s=(1,10,1)^T$ and $t=(0,1,1,2)^T$.

Find the orthogonal projection of x= $(a,b,c,d)^T$ on U. (that is, find a formula for $Proj_U(x)$). (10 points)