淡江大學九十三學年度轉學生招生考試試題 11-1

系別：數學系二年級　　　　科目：線 性 代 數

准帶項目請打「○」否則打「x」	
X	簡單型計算機

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1. 20% For the matrix

$$M = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues and eigenvectors.

(b) What is the answer of $S^{-1}MS$, where S is the matrix whose columns are the eigenvectors of M.

2. 30% For the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}.$$

(a) Find a basis for the nullspace $N(A)$ and a basis for the rowspace $\Re(A^T)$

(b) Show that nullspace $N(A)$ and the rowspace $\Re(A^T)$ are orthogonal.

(c) Prove that the nullspace and the rowspace of any matrix are orthogonal.

3. 20% Suppose A is a symmetric $n \times n$ matrix with different eigenvalues $\lambda_1, \cdots, \lambda_n$ and normalized eigenvectors $x_1, \cdots, x_n$.

(a) Show that $x_i^T x_j = 0$, $\forall i \neq j$.

(b) Prove that $A = \lambda_1 x_1 x_1^T + \lambda_2 x_2 x_2^T + \cdots + \lambda_n x_n x_n^T$.

4. 20% For the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

We can find a nonsingular matrix M such that A is similar to J, where $J = M^{-1}AM$ and J is in Jordan form. What is the matrix J and M ? (Hint: Solve AM = MJ for columns of M. You can find two columns of M in the columns of A.)

5. 10% Let A be a $m \times n$ matrix with rank n and column space $\Re(A)$. For any vector b in R^m, we say that the projection of b onto the column space $\Re(A)$ is Ax if the Euclidean distance $\|b - At\|$ for any t in R^n is minimized when $t = x$.

(a) Find the solution x such that Ax is the projection of b onto $\Re(A)$. (Hint: Differentiate $(b - Ax)^T(b - Ax)$ with respect to the vector x to have an equation.)

(b) The projection Ax in (a) can be represented as Hb where H is a $m \times m$ projection matrix, argue that $H^2 = H$, and then show the eigenvalues of H are 0's or 1's. (Hint: The projection of Ax onto $\Re(A)$ is itself.)