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淡江大學九十二學年度轉學生招生考試試題

系別：數學系二年級　　科目：線 性 代 數

准帶項目請打「○」否則打「x」	
X	簡單型計算機

Show all your work.

20 points each.

1. Let $M = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 5 & 1 & 1 \\ 1 & 1 & 2 & 5 \end{bmatrix}$. Find the rank and the determinant of M by reducing it to a row-echelon form.

2. Let $M = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & -2 \end{bmatrix}$. Find the eigenvalues and the corresponding eigenspaces of M.

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (9x+2y+z, 9x+y+z, x+y+9z)$.
(a) Find the matrix M of T with respect to the standard basis of $\mathbb{R}^3$.
(b) Let $\mathcal{B} = \{(1,1,0), (1,0,1), (0,1,1)\}$. Show that $\mathcal{B}$ is a basis of $\mathbb{R}^3$.
(c) Find an invertible matrix P such that the matrix of T with respect to the basis $\mathcal{B}$ is $P^{-1}MP$.

4. Let $\{u, v\}$ be linearly independent in a vector space V, and let $u' = au+bv$, $v' = cu + dv$, where $a, b, c, d \in \mathbb{R}$. Prove that $\{u', v'\}$ is linearly independent if and only $ad - bc \neq 0$.

5. Let $T : U \to V$ be a linear transformation of vector spaces.
(a) Prove that the kernel $ker(T)$ of T is a subspace of U.
(b) Prove that the image $im(T)$ of T is a subspace of V.
(c) Prove: $\dim(U) = \dim\big(ker(T)\big) + \dim\big(im(T)\big)$.