淡江大學九十一學年度日間部轉學生招生考試試題

系別：數學系二年級　　　　科目：線 性 代 數

准帶項目請打「○」否則打「×」
計 算 機

20% 1. Let **V** be a vector space, and suppose that **T** and **U** are linear operators on **V** such that
(a) **U** is onto.
(b) The null spaces of **T** and **U** are finite-dimensional.
Then the null space of **TU** is finite-dimensional, and

$$dim(\mathbf{N(TU)}) = dim(\mathbf{N(T)}) + dim(\mathbf{N(U)}).$$

20% 2. Show that if **W** is a finite-dimensional subspace of an inner product space **V**. Then $\mathbf{V} = \mathbf{W} \oplus \mathbf{W}^{\perp}$.

20% 3. Let $\mathbf{V} = \mathbf{R}^3$, and let $y_1 = (1,1,0)$, $y_2 = (2,0,1)$, and $y_3 = (2,2,1.)$ Then
(a) Find the orthonormal basis β of $\{y_1, y_2, y_3\}$.
(b) Find the coefficients of $x = (2,1,3)$ in the orthonormal basis β.

20% 4. Find a Jordan canonical form for A from the given data.
(1) Suppose that A is an 8×8 matrix, $A - I$ has nullity 2, $(A - I)^2$ has nullity 4, $(A - I)^k$ has nullity 5 for $k \geq 3$, and $(A + 2I)^j$ has nullity 3 for $j \geq 1$.
(2)

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{pmatrix} = SJS^{-1},$$

where S is a Jordan basis and J is a Jordan canonical form.

20% 5.(a) If $\mathbf{T} : \mathbf{P_2(R)} \longrightarrow \mathbf{P_3(R)}$ by $\mathbf{T}(f)(x) = 2f'(x) + \int_0^x 3f(t)dt$. Then
(1) Is **T** one-to-one?(If yes, prove it. Otherwise, explain your answer)
(2) Is **T** onto?(If yes, prove it. Otherwise, explain your answer)
(b) If $\mathbf{T} : \mathbf{P_2(R)} \longrightarrow \mathbf{M_{2 \times 2}}$ by

$$\mathbf{T}(f) = \begin{pmatrix} f(1) - f(2) & 0 \\ 0 & f(0) \end{pmatrix}.$$

Then
(1) Is **T** one-to-one?(If yes, prove it. Otherwise, explain your answer)
(2) Is **T** onto?(If yes, prove it. Otherwise, explain your answer)