

## 淡江大學九十學年度日間部轉學生招生考試試題

系別：數學系二年級

科目：線性代數

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本試題共 1 頁

1. Let  $A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & -2 & 2 & 4 \\ 1 & -1 & 0 & 3 \end{bmatrix}$  be  $3 \times 4$  matrix.

- (1) Find an invertible matrix  $P$  such that  $PA=R$  is a reduced row-echelon matrix. (15%)
- (2) Show that  $AX=Y$  is consistent for all  $3 \times 1$  matrix  $Y$ . (8%)
- (3) Find a basis  $B$  for the column space of  $A$  such that  $B$  is a subset of columns of  $A$ . (7%)

2. Let  $V$  be the space consisting of all polynomials of degree less than or equal 2 and the zero polynomial. Let  $B = \{x^2, x, 1\}$ . Let  $T$  be the linear transformation from  $V$  to  $V$  defined by  $T(p(x)) = p(x-1)$ . Find the matrix of  $T$  with respect to the basis  $B$ . (10%)

3. Let  $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ .

- (1) Find characteristic polynomial of  $A$  and the minimal polynomial of  $A$ . (10%)
- (2) Find an invertible matrix  $P$  such that  $P^{-1}AP = D$  is a diagonal matrix. (12%)
- (3) Let  $f(t) = (t-2)(t-3)q(t) + 2t - 3$ , where  $q$  is a polynomial. Find  $f(A)$ . (8%)

4. Let  $V$  and  $W$  be nonzero vector spaces. Prove or disprove the following statements. (20%)

- (1) If  $W \subseteq V$  and  $B$  is a basis of  $V$ , then there is a basis  $S$  of  $W$  such that  $S \subseteq B$ .
- (2) If  $A$  is a square matrix and  $A^2 = 0$ , then  $A = 0$ .
- (3) If  $Z, W$  and  $Z \cup W$  are subspaces of  $V$ , then either  $Z \subseteq W$  or  $W \subseteq Z$ .
- (4) If  $T$  is a linear transformation from  $V$  to  $V$ , then  $\ker(T)$  (the kernel of  $T$ ) is a subspace of  $V$ .

5. Let  $L$  be the line:  $x=y=z$  in 3-space. Find the orthogonal projection onto  $L$ . (10%)