

淡江大學八十九學年度日間部轉學生招生考試試題

13

系別：數學系二年級

科目：線性代數

本試題共 1 頁

1. Let \mathcal{M} be the set consisting of $m \times n$ matrices with real entries.
 - (a) Prove that \mathcal{M} is a vector space over \mathbb{R} . (5%)
 - (b) Give a basis for \mathcal{M} and find the dimension of \mathcal{M} . (5%)

2. Let A be an $n \times n$ matrix with real entries. Let $W \subset \mathbb{R}^n$ be the null space of A , and let $c \in \mathbb{R}^n$ be a particular solution to the system $AX = B$. Prove that $c + W$ is the complete set of solutions to $AX = B$. (10%)

3. Let $T : U \rightarrow V$ and $S : V \rightarrow W$ be linear transformations. Prove that the composition $S \circ T : U \rightarrow W$ is again a linear transformation. (10%)

4. Prove or disprove each of the following statements.
 - (a) If W and U are both subspaces of a vector space V , then $W \cap U$ is also a subspace of V . (5%)
 - (b) If W and U are both subspaces of a vector space V , then $W \cup U$ is also a subspace of V . (5%)
 - (c) $\det(A + B) = \det(A) + \det(B)$, where $\det(A)$ denotes the determinant of a square matrix A . (5%)

5. Let $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be given by
$$S(x, y) = (x + y, 2y),$$
$$T(x, y) = (x - y, 2x, 2y).$$
 - (a) Find the matrix representations of S , T and $T \circ S$ relative to the standard basis. (6%)
 - (b) Find the matrix representations of S relative to the basis $\{(1, 1), (0, 2)\}$. (9%)
 - (c) Evaluate $S^{100}(1, -1)$. (10%)

6. Let $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ -1 & 0 & 2 & 1 \end{bmatrix}$.

- (a) Find the characteristic polynomial and the minimal polynomial of A . (10%)
- (b) Find the eigenvalues and the corresponding eigenspaces of A . (10%)
- (c) Find the Joden canonical form J for A , and find a matrix P such that $P^{-1}AP = J$. (10%)