淡江大學八十八學年度日間部轉學生招生考試試題

系別：數學系二年級　　　　科目：線性代數

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1. (15%) Definition: Let V be a vector space and let $S = \{v_1, v_2, \cdots, v_k\}$ be vectors in a vector space K. The set of all linear combination of $v_1, v_2, \cdots, v_k$ is called the span of $v_1, v_2, \cdots, v_k$ and is denoted by Span(S).

a. Extend the linearly independent set $S = \{(1,0,-1,0), (-1,1,0,0)\}$ to a basis in R^4.

b. Find a basis from S for Span(S), where

$$S = \{(1,-1,2,3), (-2,2,-4,-6), (2,-1,6,8), (1,0,4,5), (0,0,0,1)\}$$

2. (30%) Prove or disprove the following statement.

a. Let $T : R^2 \to R^2$ be a linear transformation. If $\{v_1, v_2\} \subset R^2$ is linear dependent, then $\{T(v_1), T(v_2)\}$ is linearly dependent.

b. Let $T : R^2 \to R^2$ be a linear transformation. If $\{v_1, v_2\} \subset R^2$ is linear independent, then $\{T(v_1), T(v_2)\}$ is linearly independent.

c. Let V and W be vector spaces and $T : V \to W$ be a linear transfromation. If $\lambda_1 \neq \lambda_2$ are eigenvalues of T, and v_1, v_2 are eigenvector corresoponding to λ_1 and λ_2, respectively, then v_1, v_2 are linear independent.

3. (20%) Let P_n be the set consisting of all polynomials of degree less than or equal n and the zero polynomial. Let $B = \{1,\ x,\ x^2\}$, $B' = \{-x + x^2,\ 1 + x,\ x\}$. Let $T : P_2 \to P_2$ be the linear transformation defined by $T(a + bx + cx^2) = -2c + bx$.

a. Find the matrix A of T with respect to the standard basis B.

b. Find the matrix A of T with respect to the standard basis B'.

4.(15%) Let A be the following matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

Compute A^{99}.

5.(20%) Find a Jordan canonical form for the matrix

$$\begin{bmatrix} 4 & 0 & -1 & -1 \\ -4 & 2 & 2 & 2 \\ 2 & 1 & 2 & 0 \\ 2 & -1 & -2 & 0 \end{bmatrix}.$$