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淡江大學八十七學年度日間部轉學生入學考試試題

系別：數學系二年級　　　　　　科目：線性代數

本試題共 1 頁

1. Find the kernel of the linear transformation $T: R^3 \to R^3$ defined by $T([x,y,z]) = [3x - y + z, -2x + 2y - z, 2x + y + 4z]$. (10%)

2. (a) Find the rank of the matrix $\begin{bmatrix} 3 & 1 & 4 & 2 \\ -1 & 0 & -1 & 0 \\ 2 & 1 & 0 & 1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$. (10%)

(b) Determine whether the matrix given in (a) is invertible. (5%)

3. Let P_n be the vector space of polynomials of degree at most n. Let $T: P_3 \to P_3$ be the linear transformation defined by $T(p(x)) = p''(x) + p'(x)$.

(a) Find the matrix representation A of T relative to $B = \{1, x, x^2, x^3\}$. (5%)

(b) Find the change-coordinates matrix from B to $B' = \{x, 1+x, x+x^2, x^3\}$. (5%)

(c) Find the matrix representation A' of T relative to B'. (5%)

4. Let $\{v_1, v_2, \cdots, v_n\}$ be a basis for a vector space V. Prove that, if $w \notin span(v_1, v_2, \cdots, v_{n-1})$, then $\{v_1, v_2, \cdots, v_{n-1}, w\}$ is also a basis for V. (10%)

5. Let A and B be both $n \times n$ matrices. Prove that the product AB is invertible if and only if both A and B are invertible. (10%)

6. Let $A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ -2 & 0 & 0 & 1 \end{bmatrix}$.

(a) Find the characteristic polynomial of A. (5%)

(b) Find the real eigenvalues and the corresponding eigenspaces of A. (10%)

(c) Determine whether the matrix A given in (a) is diagonalizable; if yes, please diagonalize it. (5%)

7. Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Find a Jorden basis and the corresponding Jorden canonical form for A. (20%)