淡江大學 104 學年度日間部轉學生招生考試試題

30

系別：數學學系三年級　　科目：機率與統計學

考試日期:7 月 26 日(星期日) 第 3 節　　本試題共 6 大題， 1 頁

1. (10%) Let A_1, A_2, A_3 be independent event with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, respectively. Find $P(A_1 \cup A_2 \cup A_3)$
2. (20%)Consider the bivariate density function f (x, y) = c(x^2 + xy), $0 \le x \le 1, 0 \le y \le 1$
   (1) Find c=?
   (2) Find the conditional density of Y given X
   (3) Find Var(Y|X=x) .
3. (15%) Suppose that the moment generating function of a random variable X is given by
   $M(t) = \frac{e^{2t}}{3 - e^{3t}}, \quad t < \frac{\ln 3}{3}$.
   (1) Find the probability mass function.
   (2) Find Var(X).
4. (20%) Let X_1, X_2 be random sample of size 2 from a distribution with pdf
   $$f(x,\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & 0 < x < \infty \\ 0 & elsewhere \end{cases}$$
   (1) Find the pdf of the random variable $Y = X_1 + X_2$
   (2) It is desired to test the simple hypothesis $H_0 : \theta = 2$ against the alternative composite hypothesis $H_1 : \theta > 2$. The critical region is $C = \{(x_1, x_2) : 9.5 \le x_1 + x_2 < \infty\}$.
   Find size of the test and the probability of type II error of the test when $\theta = \theta_0 > 2$
5. (20%) A random experiment that result in a success with probability θ, $0 < \theta < 1$, and a failure with probability $1 - \theta$ is called a Bernoulli experiment. If the statistician had decided to take as many observations as needed to get the first success. Let Y denote the number of needed observations. Base on Y,
   (1) Find the maximum likelihood estimator (m.l.e.) of $1/\theta$. Is the m.l.e. an unbiased estimator of $1/\theta$?
   (2) Find the m.l.e. of $P(X > k)$.
6. (15%) Let $X_1, X_2, ..., X_n$ be random sample of size n, $n > 1$, from a distribution that is $B(1, p)$.
   (a) Find approximate $100(1-\alpha)\%$ confidence interval for p
   (b) How large a sample is required to estimate p so that with 95% confidence the maximum error of estimate of p is 0.04 (p is unknown)