

淡江大學 103 學年度日間部轉學生招生考試試題

系別： 數學系資統組三年級

科目： 機率與統計學

考試日期：7月20日(星期日)第3節

本試題共 大題 ， 1 頁

注意事項：請詳列過程，否則不予計分。

1. (10%) If $E(X^r) = 5^r, r = 1, 2, \dots$, find the moment generating function $M(t)$ of X and the probability mass function of X .
2. (10%) Assume that a policyholder is four times more likely to file exactly two claims as to file exactly three claims. Assume also that the number X of claims of this policy holder is Poisson. Determine the expectation $E(X^2)$.
3. (20%) If the moment-generating function of X is $M(t) = e^{90t+112.5t^2}$, find
 - a) (10%) the distribution of $Y = 2X - 180$,
 - b) (10%) the distribution of Y^2 and $Var(Y^2)$.
4. (20%) Let X have a uniform distribution on the interval $(0, 1)$. Given that $X = x$, let Y have a uniform distribution on the interval $(0, x)$.
 - a) (10%) Find $E(Y|x)$.
 - b) (10%) Find the marginal probability density function of Y .
5. (15%) A company provides earthquake insurance. The premium X is modeled by the probability density function

$$f(x) = \frac{x}{5^2} e^{-\frac{x}{5}}, 0 < x < \infty,$$

while the claims Y have the probability density function

$$g(y) = \frac{1}{5} e^{-\frac{y}{5}}, 0 < y < \infty.$$

If X and Y are independent, find the probability density function of $Z = X/Y$.

6. (10%) Let $f(x; \theta) = (1/\theta)x^{(1-\theta)/\theta}, 0 < x < 1, 0 < \theta < \infty$. Find the maximum likelihood estimator of θ .
7. (15%) Let X_1, X_2, \dots, X_n be a random sample of size n from an exponential distribution with unknown mean of $\mu = \theta$.
 - a) (10%) What is the distribution of the random variable $W = (2/\theta) \sum_{i=1}^n X_i$?
 - b) (5%) Use W to construct a $100(1 - \alpha)\%$ confidence interval for θ .