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淡江大學102學年度日間部轉學生招生考試試題

系別: 數學學系三年級　　　　科目: 機率與統計學

考試日期:7月24日 (星期三) 第3節　　　　本試題共 6 大題, 1 頁

注意事項: (1) 請按題號順序作答。(2) 可用鉛筆。(3) 不可使用計算機。(4) 需要計算過程。

1. (15%) A fair die is thrown twice. A is the event "sum of the throws equals 4," B is "at least one of the throws is a 3."
   (a) Calculate $P(A|B)$.
   (b) Are A and B independent events?

2. (15%) Let X and Y be two independent Bernoulli($p=\frac{1}{2}$) random variables. Define random variables U and V by: $U = X + Y$ and $V = |X - Y|$.
   (a) Determine the joint and marginal probability distributions of U and V.
   (b) Find out whether U and V are dependent or independent.

3. (15%) Suppose we choose arbitrarily a point from the square with corners at (2,1), (3,1), (2,2), and (3,2). The random variable X is the area of the triangle with its corners at (2,1), (3,1), and the chosen point. Compute $E[X]$.

4. (20%) Let $X_1, \cdots, X_n$ be a random sample from the uniform distribution over the interval I.
   (a) IF $I = [\alpha, \beta]$ (with α and β unknown, $\alpha < \beta$). Find the maximum likelihood estimates (MLEs) for α and β.
   (b) IF $I = [\theta - 1, \theta + 1]$. Find the MLE for θ.

5. (20%) Let X be a random variable with mean μ and finite variance σ^2. We want to test the hypotheses

$$H_0 : \mu = \mu_0 \text{ versus } H_1 : \mu > \mu_0,$$

   where μ_0 is specified. Let $X_1, \cdots, X_n$ be a random sample from the distribution of X and denote the sample mean and variance by $\bar{X}$ and S^2, respectively.
   (a) Find the decision rule with the approximate size α of the test.
   (b) Find the approximate power function of the test.

6. (15%) Suppose we have a dataset $x_1, \cdots, x_n$ that may be modeled as the realization of a random sample $X_1, \cdots, X_n$ from an $Exp(\lambda)$ distribution, where λ is unknown. Let $S_n = X_1 + \cdots + X_n$. Construct a 90% confidence interval for λ when $n = 20$. (The quantiles of the $Gamma(20, 1)$ distribution are $q_{0.05} = 13.25$ and $q_{0.95} = 27.88$. Denote the value of the sample mean by $\bar{x}_{20}$).