淡江大學 101 學年度轉學生招生考試試題

系別：數學學系三年級　　　　　　科目：機率與統計學

考試日期：7 月 17 日(星期二) 第 3 節　　　本試題共　9　大題，　　共　2　頁

請詳列過程，否則不予計分，共 100 分

1. (10%) Suppose discrete random variable X has the following probability mass function (pmf)

$$p(x) = \begin{cases} cx & x = 1, 2, ..., 10 \\ 0 & \text{elsewhere,} \end{cases}$$

   Find $c = ?$

2. (10%) Let continuous random variable X have the following probability density function (pdf)

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

   Find the pdf of $Y = -2\log(X)$?

3. (15%) Let random vector (X_1, X_2) have the following joint pdf

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

   (1) Find the marginal pdf of X_1.

   (2) Find $P\left(X_1 \le \dfrac{1}{2}\right)$.

   (3) Find $P(X_1 + X_2 \le 1)$.

4. (15%) Let random vector (X, Y) have the following joint pdf

$$f(x, y) = \begin{cases} 2 & 0 < x < 1, 0 < y < x \\ 0 & \text{elsewhere,} \end{cases}$$

   (1) Find the conditional density function of Y given X.

   (2) Find $E(Y \mid X = x)$ for $0 < x < 1$.

   (3) Find $E(E(Y \mid X))$.

5. (10%) Let C_1, C_2, C_3 be independent event with probabilities $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}$, respectively. Compute $P(C_1 \cup C_2 \cup C_3)$.

6. (10%) The random variable X follows a gamma distribution with the following pdf

$$f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha) \beta^\alpha}, \quad x > 0, \ \alpha > 0, \ \beta > 0.$$

Please verify the moment generating function (m.g.f.) of X is given by

$$M_X(t) = E(e^{tX}) = \frac{1}{(1-\beta t)^\alpha}, \quad t < \frac{1}{\beta}.$$

7. (10%) Let $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} N(\theta_1, \theta_2)$, where θ_1 and θ_2 are unknown, and its pdf is

$$f(x_i) = \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x_i - \theta_1)^2}{2\theta_2}\right), \quad -\infty < x_i < \infty, -\infty < \theta_1 < \infty, 0 < \theta_2$$

Find the maximize likelihood estimator (mle) of (θ_1, θ_2).

8. (10%) Let $\overline{X} = 81.2$ be the mean of a random sample of size $n = 20$ from a distribution that is $N(\mu, \sigma^2 = 80)$, where μ is unknown. Find a 95% confidence interval for μ.

Hint: $z_{0.05/2} = 1.96$.

9. (10%) Find the following union $C_1 \bigcup C_2$ and the intersection $C_1 \bigcap C_2$ of the two sets C_1 and C_2.

(1) $C_1 = \{0, 1, 2\}, C_2 = \{2, 3, 4\}$.

(2) $C_1 = \{x : 0 < x < 2\}, C_2 = \{x : 1 < x < 3\}$.