淡江大學 100 學年度轉學生招生考試試題

系別：數學學系三年級　　科目：機率與統計學

考試日期：7 月 19 日(星期二) 第 3 節　　本試題共　6 大題，一頁

(1) (21%)Let random variable Y have density as follows

$$f(y)=\begin{cases} 0.2, & -1<y\le 0 \\ 0.2+ay, & 0<y\le 1 \\ 0, & otherwise \end{cases}$$

(a) Find the value of a. (b) Find the c.d.f. F(y). (c) Find P(-0.1<Y<0.5)。

(2) (8%)Box 1 contains 2 black balls and 3 white balls, box 2 contains 2 black balls and 2 white balls, box 3 contains 3 black balls and 1 white ball. A box is chosen at random and then a ball is chosen at random from it. If the chosen ball is white, find the probability that it was from box 3.

(3) (21%)Let X_1, X_2 have joint density $f(x_1,x_2) = 6(1- x_2)$, $0 \le x_1 \le x_2 \le 1$, Find
(a) $P(X_1 \le 1/2, X_2 \le 3/4)$。(b) marginal density of X_2。(c) Are X_1 and X_2 independent(state reason)?

(4) (10%)Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability function $f(x;p) = (1-p)^{x-1}p$, x = 1,2,3,... Find the m.l.e. of p.

(5) (20%)Let $X_1, X_2, \ldots, X_{12}$ be a random sample from the Bernoulli distribution, such that $P(X_i = 1) = p$ and $P(X_i = 0) = 1-p$ for $i = 1,2,\ldots,12$ Let $Y= X_1+X_2+\ldots+X_{12}$
Suppose we want to test H_0 : p = 0.7 vs H_1 : p > 0.7, and decide to reject H_0 when $Y \ge 11$
Find (a) Type I error (b) Type II error when p = 0.9.（寫出算式即可，不用算出答案）

(6) (20%) $\bar{X}_1$ and $\bar{X}_2$ are the sample means of independent random samples of sizes n_1 and n_2 taken from normal populations with known variances σ_1^2, σ_2^2 respectively.
(a) Give the formula of a $100(1-\alpha)\%$ confidence interval for the difference of means $\mu_1 - \mu_2$.
(b) If σ_1^2, σ_2^2 are unknown but equal, $\sigma_1^2 = \sigma_2^2 = \sigma^2$, what would be an appropriate estimator of σ^2?