

淡江大學 99 學年度轉學生招生考試試題

系別：數學學系資統組三年級

科目：機率與統計學

本試題共 7 大題， / 頁

- 1 (10%) There are three independent events A, B and C. If $P(A)=0.3$, $P(B)=0.4$ and $P(C)=0.2$ then $P(A \cup B \cup C)=?$
- 2 (15%) In a certain assembly plant, three machines, B_1 , B_2 , and B_3 , make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.
- (a) What is the probability that it is defective?
 (b) If the product is defective, what is the probability that it was made by machine B_3 .
- 3 (20%) Let X and Y be the random variables with joint probability density function
- $$f(x, y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 < x < 2, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$
- (a) Find $\text{Cov}(2X-Y, X+Y/2)=?$
 (b) Find the joint distribution of $S=X+Y$ and $T=X-Y$.
 (c) Find $E(X|Y=y)$ and $\text{Var}(X|Y=y)$
- 4.(15%) Let X_1, X_2 be a random sample from $U(\theta, \theta+1)$ and we want to test $H_0: \theta = 0$ against $H_1: \theta > 0$. If a test with critical region $\{(x_1, x_2) | x_1 + x_2 \geq C\}$ is size 0.25 test then
- (a) What is the value of C?
 (b) What is the p-value if we observe that $x_1 = 0.1$ and $x_2 = 0.15$.
 (c) What is the probability of type II error when $\theta = 0.5$.
- 5.(20%) Let X_1, X_2, \dots, X_{10} and Y_1, Y_2, \dots, Y_8 be two independent random sample from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, respectively. If we want to test $H_0: \mu_1 - \mu_2 = 0$ against $H_1: \mu_1 - \mu_2 \neq 0$.
- (a) What is a test statistic for this hypotheses testing problem if σ^2 is known?
 (b) What is the distribution of the test statistic in part (a) when H_0 is true?
 (c) What is a test statistic for this hypotheses testing problem if σ^2 is unknown?
 (d) What is the distribution of the test statistic in part (c) when H_0 is true?
6. (10%) Let X_1, X_2, \dots, X_{20} be a random sample from $N(\mu, \sigma^2)$.
- (a) What is the distribution of sample variance.
 (b) Find a 95% confidence interval for the variance.
- 7.(10%) Suppose that there are n trials x_1, x_2, \dots, x_n from a Bernoulli process with parameter p, the probability of success.
- (a) Find the maximum likelihood estimator(mle) for the parameter p.
 (b) Find the variance of mle of the parameter p.