淡江大學98學年度轉學生招生考試試題

系別：數學學系資統組三年級　　　　科目：機率與統計學

准帶項目請打「V」	
X	計算機

本試題共 6 大題， 1 頁

1. (15%) The life time (in years) of a kind of light bulb has the exponential distribution $f(t) = \frac{1}{2}e^{-t/2}$.

(i) Show that $f(t)$ is a pdf, that is to show $\int_0^\infty f(t)dt = 1$

(ii) What are the mean and variance of the life time of this kind of light bulb.

(iii) A light bulb has been used for one year, what is the probability that the light bulb can be used for anther year, and what is the expectation of the life time of this light bulb.

2.(10%) Let X and Y be two independent Poisson random variables with mean λ_1 and λ_2. Find the conditional probability $P(X = x \mid X + Y = k)$, where $x = 0, 1, \cdots, k$.

3. (15%) Suppose that $X_1, X_2, \cdots, X_n$ are i.i.d. $U(0, \theta)$ with θ unknown, and $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$ are the ordered statistics.

(i). What is the joint pdf of $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$

(ii). Compute the probability $F(t) = P(X_{(n)} \le t)$, $t \in (0, \theta)$.

(iii). Use the distribution of $X_{(n)}$ to construct a confidence interval of θ with level $1 - \alpha$.

4.(20%) In a simple linear regression $Y_i = \alpha + \beta X_i + \epsilon_i$, $i = 1, \cdots, n$, where ϵ_i are i.i.d. $N(0, \sigma^2)$, let $\hat{\alpha}$, $\hat{\beta}$ be the least squares estimates of α and β.

(i) Prove that $\hat{\beta} = S_{XY}/S_{XX}$, $\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$, where $S_{XY} = \sum[(Y_i - \bar{Y})(X_i - \bar{X})]$, and $S_{XX} = \sum(X_i - \bar{X})^2$.

(ii) Show that $\hat{\beta}$ and $\hat{\alpha}$ are unbiased estimators of β and α.

5.(20%) Let $X_1, \cdots, X_{25}$ be iid $N(\mu_1, 225)$, $Y_1, \cdots, Y_{25}$ be iid $N(\mu_2, 400)$. Define $\theta = \mu_1 - \mu_2$. For testing $H_0 : \theta = 0$, v.s. $H_a : \theta > 0$.

(i) Find the UMP test with significance level 0.1.

(ii) If we observe that $\bar{X} - \bar{Y} = 8.2(5 * 1.64)$, what is the p-value ?

6. (20%) Let $X_1, \cdots, X_n$ be iid random variables from $N(\mu, \sigma^2)$.

(i). Show that the maximum likelihood estimator of μ and σ^2 are $\bar{X} = \frac{\sum_1^n X_i}{n}$ and $S^2 = \frac{(X_i - \bar{X})^2}{n}$, respectively.

(ii). Prove that $E(S^2) = \frac{n-1}{n}\sigma^2$.