

淡江大學 97 學年度轉學生招生考試試題

52
-1

系別：數學學系資統組三年級

科目：機率與統計學

可否使用計算機			
可	✓	否	

p.1

本試題共 7 大題， 2 頁

* 過程必須寫清楚，只寫答案不計分。

(1) (10%) Throw a pair of balanced dice (一對均勻骰子) 20 times, find the probability that the result (1,1) appears at least twice (兩個骰子都出一點的情況至少發生兩次) (答案不用化簡)。

(2)(15%) Let X_1, X_2 have joint p.f. (probability function) $f(x_1, x_2) = (x_1 + x_2)/32$, $x_1 = 1, 2$, $x_2 = 1, 2, 3, 4$, find

(a) $E(X_1)$

(b) The joint p.f. of $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$

(c) The p.f. of Y_2

(3) Let the random variable X have the p.d.f. $f(x) = e^{-x-1}$, $-1 < x < \infty$, find

(a) (10%) The moment generating function of X .

(b) (5%) $E(X^2)$.

(4) (10%) Let X_1, X_2, X_3 be independent $N(\mu, \sigma^2)$ random variables, find the distribution of

$$\frac{X_1 + X_2}{2} \quad \frac{X_1 + X_2 + X_3}{3}$$

(5) Let X_1, X_2, \dots, X_n be a random sample from a distribution with finite variance $\sigma^2 > 0$.

(a) (5%) Show that
$$\sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n X_i^2 - \frac{1}{n} \left(\sum_{i=1}^n X_i \right)^2$$

(b) (10%) Use the result of part (a) to show that
$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$
 is an unbiased estimator of σ^2 .

(6) (20%) Let X_1, X_2, \dots, X_n be a random sample from a $N(\mu, \sigma^2)$ distribution, and we want to test

$H_0: \mu = 0$ against $H_1: \mu = 1$.

(a) What is a test statistic (檢定統計量) for this hypotheses testing problem if σ^2 is known?

(b) What is the distribution of the test statistic in part (a) when H_0 is true?

(c) What is a test statistic for this hypotheses testing problem if σ^2 is unknown?

(d) What is the distribution of the test statistic in part (c) when H_0 is true?

(7) (15%) A survey is conducted to find out if men and women hold different opinions against a certain issue. 200 people are randomly selected (隨機抽取了200位居民做訪問), among them 100 are men and 100 are women. Among the 100 men, 40 say yes, 55 say no and 5 have no opinion; Among the 100 women, 40 say yes, 45 say no and 15 have no opinion. Use a chi-square test to determine if sex and attitude towards the issue are independent (用卡方檢定來判斷，性別和對該議題的態度是否有關). Use $\alpha = 0.05$.

本試題雙面印製

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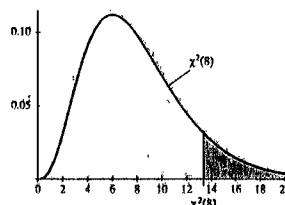
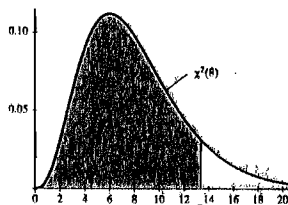
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本試題共 7 大題， 2 頁

Table IV: The Chi-Square Distribution



$$P(X \leq x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw$$

r	P(X ≤ x)							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
	χ ² _{0.99} (r)	χ ² _{0.975} (r)	χ ² _{0.95} (r)	χ ² _{0.90} (r)	χ ² _{0.10} (r)	χ ² _{0.05} (r)	χ ² _{0.025} (r)	χ ² _{0.01} (r)
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58
16	5.812	6.908	7.962	9.312	23.54	26.30	28.84	32.00
17	6.408	7.564	8.672	10.08	24.77	27.59	30.19	33.41
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.80
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.28	43.77	46.98	50.89
40	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
60	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
70	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4
80	53.34	57.15	60.39	64.28	96.58	101.9	106.6	112.3

This table is abridged and adapted from Table III in *Biometrika Tables for Statisticians*, edited by E.S.Pearson and H.O.Hartley.