系別:數學學系資統組三年級

科目:機率與統計學

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- 1. (10 points) Show that a random variable is called "memoryless" if and only if it is an exponential random variable.
- 2. (10 points) Suppose that airplane engines will fail, when in flight, with probability (1-p) independently from engine to engine. If an airplane needs a majority of its engines operative to make a successful flight, for what values of p is a 4-engine plane preferable to a 2-engine one?
- 3. (20 points) Suppose that the number of events occurs in a small interval follows from a Poisson process with parameter ②. Let X denote the waiting time until the α^{th} event occurs, find the distribution function of X and its moment generating function.
- 4. (15 points) Let (X, Y) be a random vector with p.d. f.

$$f(x,y) = \begin{cases} c, & \text{if } x^2 + y^2 \le R^2 \\ 0, & \text{elsewhere} \end{cases}$$

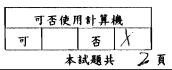
- (a) Find the value of c
- (b) Find the marginal p.d.f. of X.
- (c) Find $P(\sqrt{X^2 + Y^2} \le a)$
- 5. (15 points) Let the random vector (X, Y) has a bivariate normal distribution. Show that <u>the best linear predictor</u> of Y with respect to X is given by E[Y|X].

[Note]: For any real-valued functions, g and h, we say that the predictor g(X) is a better estimator of Y than h(X) does, if the m. s. e. of g(X) is less than that of h(X), that is $E[Y-g(X)]^2 \le E[Y-h(X)]^2$.

淡江大學 96 學年度轉學生招生考試試題

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- 6. (15 points) Consider a uniform distribution $\mathrm{U}(\theta,2\theta)$.
 - (a) Find the maximum likelihood estimator of θ .
 - (b) Does that MLE of θ converges to θ in probability (it is a consistent estimator)? Prove it, if it does.
- 7. (15 points) Consider the two independent distributions $N(\mu_1,400)$ and $N(\mu_2,225)$. Let $\theta=\mu_1-\mu_2$. For testing $H_0:\theta=0$ vs $H_A:\theta>0$, let $\overline{x},\overline{y}$ denote the observed means of two independent random samples, each of size 25 from these two distributions. Find a UMP test with significant level $\alpha=0.05$.