

系別：數學學系資統組三年級

科目：機率與統計學

可否使用計算機			
可		否	X

本試題共 2 頁

P.1

- (10 points) Show that a random variable is called "memoryless" if and only if it is an exponential random variable.
- (10 points) Suppose that airplane engines will fail, when in flight, with probability $(1-p)$ independently from engine to engine. If an airplane needs a majority of its engines operative to make a successful flight, for what values of p is a 4-engine plane preferable to a 2-engine one?
- (20 points) Suppose that the number of events occurs in a small interval follows from a Poisson process with parameter λ . Let X denote the waiting time until the α^{th} event occurs, find the distribution function of X and its moment generating function.
- (15 points) Let (X, Y) be a random vector with *p. d. f.*

$$f(x, y) = \begin{cases} c, & \text{if } x^2 + y^2 \leq R^2 \\ 0, & \text{elsewhere} \end{cases}$$

- Find the value of c
 - Find the marginal *p. d. f.* of X .
 - Find $P(\sqrt{X^2 + Y^2} \leq a)$
- (15 points) Let the random vector (X, Y) has a bivariate normal distribution. Show that the best linear predictor of Y with respect to X is given by $E[Y|X]$.

[Note]: For any real-valued functions, g and h , we say that the predictor $g(X)$ is a better estimator of Y than $h(X)$ does, if the m. s. e. of $g(X)$ is less than that of $h(X)$, that is $E[Y - g(X)]^2 \leq E[Y - h(X)]^2$.

本試題雙面印製

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P.2

6. (15 points) Consider a uniform distribution $U(\theta, 2\theta)$.
- (a) Find the maximum likelihood estimator of θ .
 - (b) Does that MLE of θ converges to θ in probability (it is a consistent estimator)? Prove it, if it does.
7. (15 points) Consider the two independent distributions $N(\mu_1, 400)$ and $N(\mu_2, 225)$. Let $\theta = \mu_1 - \mu_2$. For testing $H_0: \theta = 0$ vs $H_A: \theta > 0$, let \bar{x}, \bar{y} denote the observed means of two independent random samples, each of size 25 from these two distributions. Find a UMP test with significant level $\alpha = 0.05$.