淡江大學 95 學年度轉學生招生考試試題

系別：數學學系資統組三年級　　　科目：機率與統計學

34-1

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁

1.(10%) Among 60 year-old college professors, 10% are smokers and 90% are nonsmokers. The probability of a nonsmoker dying in the next year is 0.005 and the probability for smokers is 0.05. Given that one of this group of college professors dies in the next year, what is the conditional probability that the professor is a smoker?

2. (10%) Customers arrive in a certain shop according to an approximate Poisson process at mean rate of 20 per hour. Let X denote the waiting time in minutes until the first customer arrival. Find the p.d.f. of X and then compute $E(X)$ and $\mathrm{Var}(X)$.

3. (15%) Let X_1, X_2, X_3 and X_4 be random variables from $U(0,\theta)$, and denote their order statistics by $X_{(1)}$, $X_{(2)}$, $X_{(3)}$ and $X_{(4)}$, respectively.

i). Show that the largest order statistic $X_{(4)}$ is sufficient for θ.

ii). Compute $E(X_{(4)})$.

iii). Find a function $h(\cdot)$, so that the distribution of $h(X_1)$ is exponential distribution with mean 2.

4. (15%) Let the joint p.m.f. of X and Y be

$$f(x,y) = 1/4, \quad (x,y) \in S = \{(0,0),(1,1),(1,-1),(2,0)\}.$$

i) Compute $E(X \mid Y=0)$ and $Var(X \mid Y=0)$.

ii) Find the covariance of X and Y.

iii) Prove or disprove that X and Y are independent.

5. (15%) Let X and Y be two independent random variables with $X \sim N(1,2)$ and $Y \sim N(1,4)$.

i) Show that the moment generating function of X is e^{t+t^2}.

ii) Use the moment generating functions of X and Y to show that $X+Y$ has distribution $N(2,6)$.

iii) Compute $E\{(X+Y)^3\}$.

6. (25%) Let X_1, X_2 and X_3 be i.i.d. random variables from $N(\mu, 12)$. Consider the test for the null hypothesis $H_0 : \mu = 0$ v.s. the alternative hypothesis $H_1 : \mu = 1$ with significance level 0.05.

i). Construct a critical region A so that the test has level 0.05.

ii). Find a confidence lower bound $L(X_1,X_2,X_3)$ such that $P(L(X_1,X_2,X_3) \le \mu) = 0.95$.

iii). Show that the two event "$\{X_1,X_2,X_3\} \in A$" and "$L(X_1,X_2,X_3) > 0$" are equivalent.

iv). Find the power of this test.

v). Compute the p-value if we observe $X_1 = 1, X_2 = 0.96$ and $X_3 = 1.96$.

7. (10%) In a simple linear regression $Y_i = \alpha + \beta X_i + \epsilon_i$, $i = 1,\cdots,n$, where ϵ_i are i.i.d. $N(0,\sigma^2)$.

i). Find the maximum likelihood estimates of α and β.

ii). How to test $H_0 : \beta = 0$ against $H_1 : \beta \neq 0$ at level 0.1 ?