淡江大學九十四學年度轉學生招生考試試題

系別： 數學學系資統組三年級　科目：機率與統計學

准帶項目請打「v」	
X	簡單型計算機

注意: 每題10分, 另外請寫下適當的演算過程。

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本試題共 1 頁

1. 請舉例說明何爲中央極限定理 (central limit theorem) 以及大數法則 (law of large numbers).

2. 有 A,B,C,D,E 共5個人, A 君欲將一個紙條傳送給 E 君, A 君在紙條上可能作的記號爲 "+" 或是 "-"符號, 而這個紙條將透過 A 傳給 B, B 傳給 C, C 傳給 D 而最後 D 再傳給 E, 從過去經驗知道 A 君在紙條上作 "+"的機率爲 $\frac{1}{3}$, 而 B, C, D 三人接到紙條後可能會更改符號的機率皆爲 $\frac{2}{3}$, 現在 E 君發現手上的紙條符號是 "+", 請問在此條件下 A 君當初作的記號是 "+" 的機率爲多少 ？

3. Let X and Y be two independent Possion random variables with mean λ_1 and λ_2. Find the conditional probability $P(X = x \mid X + Y = k)$, where $x = 0, 1, \cdots, k$.

4. Let X be a normal random variable with mean μ and variance σ^2. Compute the moment generating function $E[e^{tX}]$, and then compute $E(X^3)$

5. Let $Y_1 \sim U(0, 1)$, $Y_2 \mid Y_1 \sim U(0, Y_1)$, $Y_3 \mid Y_2 \sim U(0, Y_2)$, $\cdots$, $Y_n \mid Y_{n-1} \sim U(0, Y_{n-1})$, $\cdots$.

   (i) Show that Y_3 and $X_1 X_2 X_3$ has the same distribution where X_1, X_2 and X_3 are i.i.d. $U(0, 1)$ r.v's.

   (ii) Let Z be the number of $Y_1, Y_2, \cdots, Y_n, \cdots$ in $[t, 1]$, where $0 < t < 1$, show that Z has a Poission distribution with parameter $-\log t$. (Hint: you can use the fact that $-2 \log X_i \sim \chi_2^2$ which is the exponential dist. with mean 2)

6. In a simple linear regression $Y_i = \alpha + \beta X_i + \epsilon_i$, $i = 1, \cdots, n$, where ϵ_i are i.i.d. $N(0, \sigma^2)$, let $\hat{\alpha}, \hat{\beta}$ be the least squares estimates of α and β.

   (i) Prove that $\hat{\beta} = S_{XY}/S_{XX}$, $\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$, where $S_{XY} = \sum[(Y_i - \bar{Y})(X_i - \bar{X})]$, and $S_{XX} = \sum(X_i - \bar{X})^2$.

   (ii) Show that $\hat{\beta}$ and $\hat{\alpha}$ are unbiased estimators of β and α.

7 某縣的縣長認爲若該縣 超過 2/3 的民眾贊成設立賭場, 則他就向中央爭取該縣成立合法的大型賭場, 根據一項民調結果顯示在受訪 的 900 人中有 630 人贊成賭場成立. 令 π 表示該縣贊成設立賭場的實際人口比例, 利用此筆資料檢定 $H_0 : \pi \leq 2/3$ against $H_a : \pi > 2/3$

   (i) 本題的型一錯誤及型二錯誤分別爲何 ？

   (ii) 在顯著水準爲 0.05的情形下, 根據資料你將如何建議縣長.

8. Suppose that $X_1, X_2, \cdots, X_n$ are i.i.d. $U(0, \theta)$ with θ unknown, and $X_{(1)}, X_{(2)}, \cdots, X_{(3)}$ are the ordered statistics. Use $X_{(n)}$ to construct a confidence interval of θ with level $1 - \alpha$.

9. The life time (in years) of a kind of light bulb has the exponential distribution $f(t) = \frac{1}{2}e^{-t/2}$.

   (i) What are the mean and variance of the life time of this kind of light bulb.

   (ii) A light bulb has been used for one year, what is the probability that the light bulb can be used for anther year, and what is the expectation of the life time of this light bulb.

10. 有一顆骰子連續擲了300次, 一點出現 58 次, 兩點出現 38 次, 三點出現 39 次, 四點出現 56 次, 而五點及六點忘了紀錄. 請在顯著水準 0.05 下用現有的資料下檢定這顆骰子是否正常 ？ (註: $\chi_3^2(0.95) = 7.81, \chi_4^2(0.95) = 9.48, \chi_5^2(0.95) = 11.07, \chi_6^2(0.95) = 12.59$)