淡江大學九十三學年度轉學生招生考試試題 36-1

系別：數學學系數統組三年級　　　　科目：機率與統計學

准帶項目請打「○」否則打「×」	
×	簡單型計算機

節次：7月14日第3節

本試題共 2 頁

本試題雙面印製

1. We want to study the geographical distribution of a certain species of jumping mouse in a wide region. The region was divided into n small, equally sized zone, where one could either find or not find a trapped mouse. If the mice were randomly distributed around the region, but there were relatively few of them in total, it would be reasonable to model each zone as a Bernoulli trial, with small success probability P_n of finding a trapped mouse. If we envision increasing the number n of zones while decreasing zone size, it stands to reason that P_n should decrease. Let us suppose that P_n is proportional to (1/n). Let the random variable X_n denote the number of trapped mice throughout the region. (25 points)

   (a) Under what assumptions that the random variable X_n would have the binomial distribution with parameters $(n,\ P_n)$?

   (b) Show that the limiting distribution as n goes to ∞ is the Poisson probability mass function with parameter being the proportional constant as in $P_n \propto \frac{1}{n}$ 。

   (c) Let T_1 be the time of the first mouse been trapped. Find it's probability density function 。

2. Does the Cauchy density function $f(x) = \frac{1}{[\pi(1+x^2)]}$，$x \in$ have a mean? a variance? If the answer is Yes, then find it? (10 points)

3. Let X and Y be random variables with mean $\mu_x,\ \mu_y$ *and variance* σ_x, σ_y respectively, and with correlation coefficient ρ. Suppose the conditional mean of Y given X = x is a linear function of x, then find E(Y| X = x). (10 points)

◀注意背面尚有試題▶

淡江大學九十三學年度轉學生招生考試試題

系別：數學學系數統組三年級　　科目：機率與統計學

准帶項目請打「○」否則打「×」	
×	簡單型計算機

節次：7月14日第3節

4. Let independent random variables X and Y be of Uniform density function with parameter (0, 1). Let W denote the **min(X, Y)** and V denote **max(X, Y)**. (20 points)

   (a) Find the joint density function of (W,V).

   (b) What is the conditional density function of V|W=w ?

5. Let $X_1, X_2, \cdots, X_n$ be a random sample with density function $f(x;\ \theta)$.

   (a) Give the definition of a statistic $Z = g(X_1, X_2, \cdots, X_n)$ which is called sufficient for θ.

   (b) Find a sufficient statistic for the parameter θ in the distribution with density $f(x;\ \theta) = \theta x^{\theta-1};\ 0 < x < 1$.

   (c) Show that the MLE of θ is a function of the sufficient statistic and itself sufficient. (15 points)

6. Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal distribution with mean μ, and standard deviation σ. (20 points)

   (a) State the statistical meaning of a $100(1-\alpha)\%$ Confident interval for μ.

   (b) How could we use the $100(1-\alpha)\%$ C.I. for μ to test $\mu = 0$?