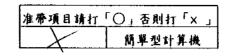
## 淡江大學九十三學年度轉學生招生考試試題 34~1

系別:數學學系數統組三年級

科目:機率與統計學



節次: 7月14日第3 節 本試題共 2 頁

- We want to study the geographical distribution of a certain species of jumping mouse in a wide region. The region was divided into n small, equally sized zone, where one could either find or not find a trapped mouse. If the mice were randomly distributed around the region, but there were relatively few of them in total, it would be reasonable to model each zone as a Bernoulli trial, with small success probability Pn of finding a trapped mouse. If we envision increasing the number n of zones while decreasing zone size, it stands to reason that Pn should decrease. Let us suppose that Pn is proportional to (1/n). Let the random variable Xn denote the number of trapped mice throughout the region.
  - (a) Under what assumptions that the random variable  $X_n$  would have the binomial distribution with parameters  $(n, P_n)$ ?
  - (b) Show that the limiting distribution as n goes to  $\infty$  is the Poisson probability mass function with parameter being the proportional constant as in  $P_n \propto \frac{1}{n}$ .
  - (c) Let  $T_1$  be the time of the first mouse been trapped. Find it's probability density function •
- 2. Does the Cauchy density function  $f(x) = \frac{1}{[\pi(1+x^2)]}$ ,  $x \in \text{have a mean? a}$  variance? If the answer is Yes, then find it? (10 points)
- 3. Let X and Y be random variables with mean  $\mu_x$ ,  $\mu_y$  and variance  $\sigma_x$ ,  $\sigma_y$  respectively, and with correlation coefficient  $\rho$ . Suppose the conditional mean of Y given X = x is a linear function of x, then find E(Y| X = x).

(10 points)

## 淡江大學九十三學年度轉學生招生考試試題 36-2

系別:數學學系數統組三年級

科目:機率與統計學

准带项目請打	「〇」否則打「× 」
X	簡單型計算機

節次: 7月14日第3節本試題共 三 頁

- 4. Let independent random variables X and Y be of Uniform density function with parameter (0, 1). Let W denote the min(X, Y) and V denote max(X, Y).

  (20 points)
  - (a) Find the joint density function of (W,V).
  - (b) What is the conditional density function of V|W=w?
- 5. Let  $X_1, X_2, \dots, X_n$  be a random sample with density function  $f(x; \theta)$ .
  - (a) Give the definition of a statistic  $Z = g(X_1, X_2, \dots, X_n)$  which is called sufficient for  $\theta$ .
  - (b) Find a sufficient statistic for the parameter  $\theta$  in the distribution with density  $f(x; \theta) = \theta x^{\theta-1}$ ; 0 < x < 1.
  - (c) Show that the MLE of  $\theta$  is a function of the sufficient statistic and itself sufficient. (15 points)
- 6. Let  $X_1, X_2, \dots, X_n$  be a random sample from a normal distribution with mean  $\mu$ , and standard deviation  $\sigma$ . (20 points)
  - (a) State the statistical meaning of a  $100(1-\alpha)\%$  Confident interval for  $\mu$ .
  - (b) How could we use the  $100(1-\alpha)\%$  C.I. for  $\mu$  to test  $\mu = 0$ ?