淡江大學九十二學年度轉學生招生考試試題

系別：數學系數統組三年級　　科目：機率與統計學

准帶項目請打「○」否則打「×」	
×	簡單型計算機

本試題共 2 頁

1. (15%) Suppose that $P(A)=\frac{2}{5}$, $P(B)=\frac{3}{5}$, $P(C)=\frac{7}{10}$, $P(A\cup B)=\frac{21}{25}$, $P(A\cup C)=\frac{39}{50}$, $P(B\cup C)=\frac{93}{100}$, $P(A\cap B\cap C)=\frac{3}{25}$. Find

a) $P(A\cap B\cap C^c)$; b) $P(A\cap B^c\cap C^c)$; c) $P(A^c\cap B^c\cap C^c)$.

2. (20%) Let X and$Y$ have the joint probability density function:

$$f(x,y)=\frac{x+y}{32},\quad x=1,2;\quad y=1,2,3,4.$$

a) Find the conditional probability density functions $g(x|y)$ and $h(y|x)$.

b) Find $P(1\le Y\le 3|X=1)$.

c) Find the conditional mean $E(X|Y=1)$.

d) Find the conditional variance $Var(X|Y=1)$.

3. (20%) Let X_1, X_2 be a random sample of size 2 from a distribution with probability density function

$$f(x;\theta)=\frac{1}{\theta}e^{-\frac{x}{\theta}},\quad 0<x<\infty;\quad 0<\theta<\infty.$$

a) Find the joint probability density function of $Y_1=X_1+X_2$ and $Y_2=X_2$.

b) Find the conditional probability density function of Y_2, given $Y_1=y_1$.

4. (15%) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the probability density function:

$$f(x;\theta)=\frac{1}{\theta^2}xe^{-\frac{x}{\theta}},\quad 0<x<\infty;\quad 0<\theta<\infty.$$

Find the maximum likelihood estimator of θ.

5. (10%) The probability density function of X is defined by $f(x)=\frac{1}{\theta}$, $0<x<\theta$. We shall test the null hypotheses $H_0:\theta=\frac{4}{3}$ against the alternative hypotheses $H_1:\theta=\frac{7}{3}$ using a single observation of X. Let the critical region be defined by $C=\{x:x>1\}$. Find

a) α, the significance level of this test;

b) β, the probability of Type II error.

6. (20%) Let $X_1, X_2, \ldots, X_{25}$ be a random sample of size $n=25$ from $N(\mu,100)$. To test H_0: $\mu=80$ against $H_1:\mu>80$. Let the critical region be defined by $C=\{(x_1,x_2,\ldots,x_{25}):\bar{x}>83.2\}$, where $\bar{x}$ is the sample mean.

a) Find the power function of this test and the power at $\theta=85$.

b) What is the p-value corresponding to $\bar{x}=78.5$.

本試題雙面印製

淡江大學九十二學年度轉學生招生考試試題 33-2

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The Normal Distribution

$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[\Phi(-z) = 1 - \Phi(z)]$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990