淡江大學九十一學年度日間部轉學生招生考試試題

系別：數學系數統組三年級　　　科目：機率與統計學

准帶項目請打「○」否則打「x」
X 計算機

本試題雙面印製

Distributions that you may need:

(1) $\underset{\sim}{X}_{n\times1} \sim MVN(\underset{\sim}{\mu}_{n\times1}, \Sigma_{n\times n})$ then the p.d.f of $\underset{\sim}{X}$ is given by

$$f(\underset{\sim}{x}) = \frac{1}{[2\pi]^{n/2}\sqrt{\det(\Sigma)}} \exp\left\{\frac{-1}{2}(\underset{\sim}{x}-\underset{\sim}{\mu})'\Sigma^{-1}(\underset{\sim}{x}-\underset{\sim}{\mu})\right\}, \quad \underset{\sim}{x} \in R^n.$$

(2) $X \sim \text{Poisson}(\lambda)$ then $f(x) = \frac{\lambda^x}{x!} e^{-\lambda}$, $x = 0,1,2,\cdots$.

(3) $X \sim \text{Gamma}(\alpha,\lambda)$ then $f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$, $x > 0$.

with moment generating function, $\left(\frac{\lambda}{\lambda - t}\right)^\alpha$, $|t| < \lambda$.

Some properties you may need:

Prop.1 $$\sum_{x=0}^{k-1} \frac{(\lambda t)^x e^{-\lambda t}}{x!} = \int_{\lambda t}^{\infty} \frac{x^{k-1}e^{-x}}{(k-1)!}\,dx.$$

Prop.2 Let $\underset{\sim}{x}_{n\times1} \sim MVN(\mu\underset{\sim}{1}_{n\times1}, \sigma^2 I_n)$, and $Q_{n\times n}$ be an n×n idempotent matrix (i.e. $Q^2 = Q$) with rank γ. If $\mu = 0$ or $Q\underset{\sim}{0}_{n\times1} = \underset{\sim}{0}_{n\times1}$ then $\underset{\sim}{x}'Q\underset{\sim}{x}/\sigma^2 \sim \chi^2(\gamma)$ (i.e. a Chi-Square distribution with γ degree of freedoms). Where $\underset{\sim}{1}_{n\times1}$ and $\underset{\sim}{0}_{n\times1}$ are n×1 column vectors of 1's and o's respectively.

Possible δ_α **- values:** $P\{Z > \delta_\alpha\} = \alpha$

α	0.1	0.05	0.025	0.01
δ_α	1.285	1.645	1.96	2.325

Noatation: $t_\alpha(n)$ is the value that $P\{T > t_\alpha\} = \alpha$, where T has t distribution with n degree of freedoms.

1. In a Poisson process with a rate of λ, let N_t denote the number of arrivals in the time interval $[0,t]$ and T_n ($n = 1, 2,\cdots$) represent the waiting time for the next n arrivals.

(a) Use the Prop.1 to show that the distribution of T_n is Gamma(n,λ). **(10 points)**

(b) Find the conditional c.d.f. (cumulative distribution function) of $(T_n - T_{n-1})$ given that $N_t = 1$ and interpret your result. **(10 points)**

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2. Let $x_1, x_2, \cdots, x_n$ be a random sample from a Normal population with mean μ_X, variance σ^2, denoted by $\{X_i\}_{i=1}^n \sim N(\mu_X, \sigma^2)$, and sample variance S_X^2. And, $\{Y_i\}_{i=1}^n \sim N(\mu_Y, \sigma^2)$, with sample variance S_Y^2. Note that μ_X, μ_Y and σ^2 are all unknown parameters.

(a) Use Prop. 2 to show that $\frac{(n-1)S_X^2}{\sigma^2} \sim \chi^2(n-1)$. (10 points)

(b) Show that S_X^2 is an unbiased estimators of σ^2. (5 points)

(c) Does S_X^2 achieve the Rao-Cramer lower bound? Justify your answer. (10 points)

(d) Find another unbiased estimator of σ^2 with variance less than the variance of S_X^2 ? Justify your answer. (10 points)

(e) Use each of S_X^2 and the estimator found in part (d) to derive two 95% confidence intervals of μ_Y. Explain why those two intervals have different interval lengths. (15 points)

(f) What is the meaning of "95%" and the terminology "confidence" in the part (e)? (10 points)

3. If an investor wants to gain information about the unknown weekly rate of return μ on a risky asset, for how many weeks should the investor observe its rates of return in order to produce an estimate that falls within a tolerance of 0.005 of the true μ, with a probability of at least 80%? Suppose that the variance of the asset rate of return is no mare than 0.0004. (10 points)

4. In testing H_0: $\theta \in \Theta_0$ against H_A: $\theta \in \Theta_A$, where Θ_0 and Θ_A are disjoint sets of possible values of the parameter θ whose union makes up the entire set of possible values for θ, assume that the significance level is 0.05. Use one or two sentences to make your conclusion when the p-value is 0.01 and the p-value is 0.1. (10 points)