淡江大學八十七學年度日間部轉學生入學考試試題

系別：數學系數理統計組三年級　　科目：機率與統計學

本試題共 1 頁

※計算過程儘量寫清楚

1. (10%) Bus tickets in a certain city contain four numbers, U, V, W and X. Each of these numbers is equally likely to be any of the ten digits $0, 1, \ldots, 9$, and the four numbers are chosen independently. A bus rider is said to be lucky if $U + V = W + X$. What proportion of the riders are lucky? (Just write down the expression without calculating the final answer.)

2. (20%) Suppose that a fair coin is tossed independently n times. Determine the probability of obtaining exactly $n-1$ heads, given a) that at least $n-2$ heads are obtained and b) that heads are obtained on the first $n-2$ tosses.

3. (20%) Suppose that the random variables X and Y have the following joint probability density function:

$$f(x,y) = \begin{cases} 8xy & \text{for } 0 \le x \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Also, let $U = X/Y$ and $V = Y$.

a) Determine the joint p.d.f of U and V.

b) Are X and Y independent? Are U and V independent?

4. (10%) Suppose that X and Y are random variables such that $E(X|Y) = 10 - Y$ and $E(Y|X) = 7 - X/4$. Determine the correlation of X and Y.

5. (10%) Suppose that a random sample of four observations is drawn from a Poisson distribution with mean λ, and let $\bar{X}$ denote the sample mean. Show that

$$P(\bar{X} < 1/2) = (4\lambda + 1)e^{-4\lambda}.$$

6. (10%) Suppose that $X_1, X_2, \ldots, X_n$ form n Bernoulli trials with parameter $\theta = (1+\beta)/3$, where the value of β is unknown $(0 \le \beta \le 1)$. Determine the maximum likelihood estimator of β.

7. (20%) Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from a uniform distribution on the interval $(0, \theta)$, and that the following hypotheses are to be tested:

$$H_0 : \theta \ge 2, \quad H_1 : \theta < 2.$$

Let $Y_n = \max(X_1, \ldots, X_n)$, and consider a test procedure such that the critical region contains all the outcomes for which $Y_n \le 1.5$.

a) Determine the power function of the test.

b) Determine the size of the test.