淡江大學 104 學年度日間部轉學生招生考試試題

系別：數學學系三年級　　科目：線 性 代 數

考試日期：7 月 26 日(星期日) 第 1 節　　本試題共 6 大題，1 頁

1. (20 %) Find the characteristic polynomial and the eigenvalues for the 3×3 matrix $A = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix}$.

2. (10 %) Find the inverse matrix B^{-1} for the 3×3 matrix $B = \begin{bmatrix} -1 & -2 & 11 \\ 1 & 3 & -15 \\ 0 & -1 & 5 \end{bmatrix}$.

3. (20 points) Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.

   (1) Find an invertible 3×3 matrix P and a 3×3 diagonal matrix D such that $P^{-1}AP = D$.

   (2) Calculate A^5.

4. (20 points) Let

$$W_1 = \left\{ \begin{bmatrix} -b \\ a \\ b+c \\ a+c \end{bmatrix} \middle| a, b, c \in \mathbb{R} \right\}, \qquad W_2 = \left\{ \begin{bmatrix} -a \\ 0 \\ 2a+b \\ a+2b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\}.$$

Find $\dim(W_1), \dim(W_2), \dim(W_1 \cap W_2)$

5. (20 points) Let V be the vector space of 2×2 matrices, and let $T : V \to V$ be defined by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} -3a+5d & 3b-5c \\ -2c & 2d \end{bmatrix}.$$

Find the matrix of T with respect to the basis $C = \{A_1, A_2, A_3, A_4\}$, where

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

6. (10 points) Let $T : V \to V$ be a one-to-one linear transformation and $\dim(V) = n$. Show that T is onto.