

淡江大學 104 學年度日間部轉學生招生考試試題

系別：數學學系三年級

科目：高等微積分

29-1

考試日期：7月26日(星期日) 第4節

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1. (10%) A sequence $\{x_n\}$ in \mathbb{R} is said to be a Cauchy sequence in case for every $\varepsilon > 0$, there is a natural number n_0 such that for all $m, n \geq n_0$, then $|x_m - x_n| < \varepsilon$. Show that a convergent sequence is a Cauchy sequence.

2. (10%) Let $a_1 = 1$ and $a_{n+1} = (2 + a_n)^{\frac{1}{2}}$ for $n = 1, 2, 3, \dots$

(a) Show that a_n is monotone increasing (by induction).

(b) Show that a_n is bounded above by 2 (by induction).

(c) Find the $\lim_{n \rightarrow \infty} a_n$.

3. (10%) Let $O_i \subset \mathbb{R}$ be an open set for $i \in \mathbb{N}$. Show that $\bigcap_{i=1}^{\infty} O_i$ is open or give a counterexample.

4. (10%) Test for convergence or divergence

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1.5}}$$

5. (10%) Classify the following series into absolute convergence, conditional convergence or divergence. Explain why.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

6. (20%) (a) Give the definition of uniform continuity of a function f defined on a set $E \subseteq \mathbb{R}$.

(b) Give the definition of a set E being "compactness" in \mathbb{R} .

(c) Show that if f is continuous on a compact set $E \subseteq \mathbb{R}$, then f is uniformly continuous.

7. (10%) Let $f_n(x) = \frac{x^2 + nx}{n}$ on \mathbb{R} .

(a) Find the limit function $f(x)$.

(b) Does $f_n(x)$ converge to $f(x)$ uniformly? Explain why.

本試題雙面印刷

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8. (10%) Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} 0 & (x, y) = (0, 0) \\ \frac{xy^2}{x^2 + y^2} & (x, y) \neq (0, 0) \end{cases}$$

(a) Find $f_x(0, 0)$ and $f_y(0, 0)$.

(b) Find the partial derivative of f at $(0, 0)$ with respect to any vector $\vec{u} = (a, b)$ is

$$D_{\vec{u}}f(0, 0) = \frac{ab^2}{a^2 + b^2}.$$

(c) Show that f is not differentiable at $(0, 0)$.

9. (10%) Evaluate

$$\int_0^4 \int_{\frac{x}{2}}^2 e^{y^2} dy dx.$$