

# 淡江大學 104 學年度日間部轉學生招生考試試題

系別：數學學系三年級

科目：代 數

考試日期：7 月 26 日(星期日) 第 3 節

本試題共 7 大題， 1 頁

Please show your work.

- (10 pts) Find all integers  $x$  satisfying  $34x \equiv 3 \pmod{97}$ .
- (20 pts) Let  $S_4$  be the symmetric group on 4 symbols.  
(a) Is there an element of order 8 in  $S_4$ ? If so, please give an example. If not, explain why not. (b) Is there a subgroup of order 8 in  $S_4$ ? If so, please give an example. If not, explain why not.
- (10 pts) Let  $G = \langle a \rangle$  be a cyclic group of finite order  $n$ . Show that  $G$  is isomorphic to the additive group  $\mathbb{Z}_n$  by constructing an isomorphism from  $G$  to  $\mathbb{Z}_n$ .
- (10 pts) Determine if the groups  $\mathbb{Z}_6 \times \mathbb{Z}_3$  and  $\mathbb{Z}_9 \times \mathbb{Z}_2$  are isomorphic or not. Please justify your answer.
- (10 pts) Let  $G = \mathbb{Z}_3 \times \mathbb{Z}_9$  and let  $H$  be the cyclic subgroup of  $G$  generated by  $(0, 2)$ . Describe the quotient group  $G/H$ . What is its order?
- (20 pts) Let  $f(x) = 5x^{10} - 6x^7 + 15x^5 - 21x^2 + 36x + 6$ .  
(a) State Eisenstein's criterion and apply it to show that  $f(x)$  is irreducible in  $\mathbb{Q}[x]$ .  
(b) Is  $f(x)$  irreducible in  $\mathbb{Z}_5[x]$ ? Justify your answer.
- (20 pts) (a) Let  $I$  be an ideal in a commutative ring  $R$ . Give a definition for  $I$  being prime. (b) Describe all prime ideals in the ring  $\mathbb{Z}$ . (c) Describe all prime ideals in the ring  $\mathbb{Z}_{20}$ .