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淡江大學 103 學年度日間部轉學生招生考試試題

系別：數學學系三年級　　　　科目：線 性 代 數

考試日期：7 月 20 日(星期日) 第 1 節　　　　本試題共 7 大題， 1 頁

1 . (15%) Assume that A is a $n\times n$ symmetric matrix. Also x and y are vectors in R^n with the properties that $Ax=\lambda_1 x$ and $Ay=\lambda_2 y$ where $\lambda_1\neq\lambda_2$. Prove that x and y are orthogonal and linearly independent

2. (15%) If $A=\left[a_{ij}\right]_{4\times4}=\begin{bmatrix}1&1&1&1\\a&a^2&a^3&a^4\\b&b^2&b^3&b^4\\c&c^2&c^3&c^4\end{bmatrix}$ and suppose the cofactor of a_{2j} is C_{2j}, j=1,2,3,4.

(1) Find det(A).

(2) Find $b\cdot C_{21}+b^2\cdot C_{22}+b^3\cdot C_{23}+b^4\cdot C_{24}$

3.(15%) Consider the bases $B=\{u_1,u_2,u_3\}$ and $B'=\{v_1,v_2,v_3\}$ for R^3 , where $u_1=(-3,0,-3)^t$, $u_2=(-3,2,-1)^t$, and $u_3=(1,6,-1)$, $v_1=(-6,-6,0)^t$, $v_2=(-2,-6,4)^t$, and $v_3=(-2,-3,7)^t$

a. Find the transition matrix from B to B'

b. Compute the coordinate matrix $[w]_B$, where $w=(-5,8,-5)^t$ and use transition matrix to compute

4. (15%) Suppose T is a linear transformation from V into W. Let $\dim(V)=n$ and $\dim(W)=m$. Suppose $S=\{v_1,\dots,v_r\}$ where $0<r<n$ is a basis of null space of T. and $S^*=S\cup\{v_{r+1},\dots,v_n\}$ is a basis of V. Show that

(a) $\{T(v_{r+1}),\dots,T(v_n)\}$ is a basis of range(T).

5. (10%)Suppose that A is a 5×4 matrix and B is a 4×5 matrix. Prove that AB is not invertible.

6 (15%) Given $A=\begin{bmatrix}0&2&-1\\2&3&-2\\-1&-2&0\end{bmatrix}$

(1) Find an orthogonal U that diagonalizes A.

(2) Find the eigenvalues and bases for the eigenspaces of A^{25}

7.(15%) Let $C[-\pi,\pi]$ be the real vector space of continuous real valued function on $[-\pi,\pi]$. For two function f, g in $C[-\pi,\pi]$ define an inner product by $<f,g>=\int_{-\pi}^{\pi}f(x)g(x)dx$

(a) Find the angle between $conx$ and $\sin x$.

(b) If we know that $S=\{\frac{1}{\sqrt{2\pi}},\frac{1}{\sqrt{\pi}}conx,\frac{1}{\sqrt{\pi}}con2x,\frac{1}{\sqrt{\pi}}con3x,\frac{1}{\sqrt{\pi}}con4x\}$ is an orthonor-mal set in $C[-\pi,\pi]$. Determine the values of $\int_{-\pi}^{\pi}\sin^4 x\cos 2x dx$. (note: $\sin^4 x=\frac{3}{8}-\frac{\cos 2x}{2}+\frac{\cos 4x}{8}$)