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淡江大學 103 學年度日間部轉學生招生考試試題

系別：數學學系三年級　　　　科目：高等微積分

考試日期：7 月 20 日(星期日) 第 4 節　　　　本試題共 7 大題， 1 頁

(1) (20%) Test for convergence or divergence. Explain your answers.

(a) $\int_0^\infty \frac{x}{1+x^3}\,dx$.

(b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$, $p > 1$.

(c) $\int_0^\infty \frac{\sin x}{x}\,dx$.

(d) $\int_0^\infty e^{-x^2}\,dx$.

(2) Evalue the integral

(a) (7%) $\int_0^2 \int_0^{\sqrt{4-x^2}} \sin(x^2+y^2)\,dy\,dx$.

(b) (8%) $\iint_E e^{\frac{y-x}{y+x}}\,dA$, where E is the trapezoid with vertices $(1,1)$, $(2,2)$, $(2,0)$ and $(4,0)$.

(3) (15%) Let $E_i \subset \mathbb{R}^n$ be an open set for $i \in \mathbb{N}$.

(a) Show that $\cup_{n=1}^{\infty} E_n$ is open.

(b) Show that $\cap_{i=1}^{n} E_i$ is open.

(c) Show that $\cap_{n=1}^{\infty} E_n$ might be not open.

(4) Let

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^4}, & x^2+y^2 \neq 0, \\ 0, & x^2+y^2 = 0. \end{cases}$$

(a) (6%) Compute $f_x(0,0)$ and $f_y(0,0)$.

(b) (4%) Show that f is not differentiable at $(0,0)$ even though both $f_x(0,0)$ and $f_y(0,0)$ exist.

(5) $a, b \in \mathbb{R}$ and $a < b$. Let $f : (a,b) \to \mathbb{R}$ be a function.

(a) (7%) Suppose that $|f(x) - f(y)| \leq M|x-y|$ for some $M \in \mathbb{R}$ and $x, y \in (a,b)$. Show that $f(x)$ is uniformly continuous.

(b) (8%) Suppose that f is uniformly continuous in (a,b). Show that f can be extened to a continuous function defined on $[a,b]$.

(6) (10%) Let $f : [a,b] \to \mathbb{R}$ be a continuous function, where $a < b$ are real numbers. Show that

$$\lim_{n\to\infty} [\int_a^b |f(x)|^n dx]^{1/n} = \max_{x\in[a,b]} |f(x)|.$$

(7) Let $g_n : [a,b] \to \mathbb{R}$ be a continuous function and $|g_n(x)| < L$ for some $L > 0$ and all $n \in \mathbb{N}$.

(a) (7%) Show that $\sum_{n=1}^{\infty} \frac{g_n(x)}{n^2}$ converges uniformly on $[a,b]$.

(b) (8%) Denote $G_n(x) = \int_a^x g_n(t)\,dt$. Show that $\{G_n(x)\}_{n=1}^{\infty}$ has a subsequence $\{G_{n_k}(x)\}_{k=1}^{\infty}$ which is convergent uniformly.