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# 淡江大學 103 學年度日間部轉學生招生考試試題

系別：數學學系三年級

科目：代 數

考試日期：7 月 20 日(星期日) 第 3 節

本試題共 9 大題， 1 頁

**Show your work.**

1. Show that every group of order 11 is cyclic. (10 pts)
2. Find the remainder when  $103^{10}$  is divided by 11. (10 pts)
3. Suppose a group  $G$  has subgroups of order 45 and 75. If  $500 \leq |G| \leq 700$ , determine  $|G|$ . (10 pts)
4. Show that  $|Z_n^*|$  is even for integer  $n \geq 3$ . (10 pts)
5. Let  $H$  and  $K$  be subgroups of a group  $G$ . (20 pts)
  - (a) Show that  $Ha \cap Ka = (H \cap K)a \quad \forall a \in G$ .
  - (b)  $\forall a, b \in G$ . Show that either  $Ha \cap Kb = \phi$  or  $Ha \cap Kb = (H \cap K)c$  for some  $c \in G$ .
6. Prove that  $\{\varepsilon, (12)(34), (13)(24), (14)(23)\}$  is a normal subgroup of  $S_4$ . (10 pts)
7. Let  $R$  be a ring,  $x \in Z(R)$ ,  $Z(R)$  is the center of  $R$ . Show that  $\text{ann}(x) = \{r \mid rx = 0\}$  is an ideal of  $R$ . (10 pts)
8. Prove that an ideal  $P \neq R$  of a commutative ring  $R$  is a prime ideal if and only if  $R/P$  is an integral domain. (10 pts)
9. Let  $R$  be a ring.  $r^3 = r$ , show that  $R$  is commutative. (10 pts)