淡江大學 102 學年度日間部轉學生招生考試試題

系別：數學學系三年級　　　　科目：高等微積分

考試日期：7 月 24 日(星期三) 第 4 節　　　　本試題共　8 大題，　一 頁

1. (10%)Show that $\frac{5}{4}$ is an isolated point of the set $\left\{(-1)^n+(-1)^n\frac{1}{n}\right\}$.

2. (10%) Let f be defined by $f(x)=\begin{cases} x^2+2\,, x\le 2 \\ ax+b\,, x>2 \end{cases}$, for what values of a and b is f differentiable at $x=2$?

3. (20%)Determine whether the following series are convergent or divergent:

(a) $\sum_{n=1}^{\infty}\frac{(-1)^n 2n^2}{100n^2+3n+1}$　(b) $\sum_{n=1}^{\infty}\frac{2\sqrt{n}}{n^2-3n+1}$

4. (10%) Let $f(x_1,x_2)=\begin{cases} \frac{x_1x_2}{x_1^2+x_2^2}, (x_1,x_2)\ne(0,0) \\ 0,\ (x_1,x_2)=(0,0) \end{cases}$, show that the partial derivatives of $f(x_1,x_2)$ with respect to x_1 and x_2 exist at the origin.

5. (10%) Evaluate $\int_0^1\int_x^1 e^{x/y}\,dy\,dx$

6. (20%)Let $f:D\to R$ be a function and $E\subseteq D$.

(a) Write the definition of f being a "uniformly continuous" function on E.

(b) If $f(x)$ is defined as

$$f(x)=\begin{cases} 2x-1, & 0\le x\le 1 \\ x^3-5x^2+5, & 1<x\le 2 \end{cases}$$

Determine if $f(x)$ is uniformly continuous on $[0,2]$. (State reasons).

7. (10%)Let $f_n(x)=\begin{cases} 2x+\frac{1}{n}, & 0\le x<1 \\ \exp\left(\frac{x}{n}\right), & 1\le x<2 \\ 1-\frac{1}{n}, & x\ge 2 \end{cases}$

Let $\lim_{n\to\infty} f_n(x)=f(x)$, find $f(x)$ and determine if $f_n(x)$ converges uniformly to $f(x)$ on $[0,\infty)$.

8. (10%) Let $f(x,y,z)=x^2+y^2+z^2$, consider the problem of finding extreme values of f subject to the condition: $2x-y+2z=16$. Use the method of Lagrange multiplier to find the stationary points and determine if they are local max, local min or saddle points.