淡江大學 101 學年度轉學生招生考試試題

系別：數學學系三年級　　　　科目：線 性 代 數

考試日期:7月17日(星期二) 第1節　　　　本試題共 7 大題， 1 頁

1. (13%) Let $A=\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ -6 & 2 & 3\end{bmatrix}$. Find the inverse of A and the determinant of $(2A)^{-1}$.

2. (12%) Determine whether the followings are linearly dependent or linearly independent.
   (a) $v_1=(1,1,-2), v_2=(2,5,-1), v_3=(0,1,1)$.
   (b) $v_1=\begin{bmatrix}2 & 1\\ 0 & 1\end{bmatrix}, v_2=\begin{bmatrix}3 & 0\\ 2 & 1\end{bmatrix}, v_3=\begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix}$.

3. (21%) Let $A=\begin{bmatrix}3 & -2 & 4\\ -2 & 6 & 2\\ 4 & 2 & 3\end{bmatrix}$.

   (a) Find the characteristic polynomial, eigenvalues, and corresponding eigenvectors of A.
   (b) Find an invertible matrix P and diagonal matrix D such that $D=P^{-1}AP$.

4. (12%) Let $u_1=(1,1,1), u_2=(0,1,1), u_3=(0,0,1)$. Consider the standard inner product on $\mathbb{R}^4$.

   Apply the Gram-Schmidt process to transform $\{u_1,u_2,u_3\}$ into an orthonormal basis.

5. (16%) Let $T: P_2(\mathbb{R})\to P_2(\mathbb{R})$ defined by $T(f)=f''-3f'+4f$. Here $P_2(\mathbb{R})$ is the set of all polynomials with degree ≤ 2.
   (a) Find the matrix representation in the basis $\{1,x,x^2\}$.
   (b) Find the range and the null space of T.
   (c) Find the nullity and rank of T.

6. (10%) Let A and B be $n\times n$ matrices such that $A+B=AB$. Prove that $AB=BA$.

7. (16%) Let W be a finite-dimensional subspace of an inner product space $(V,<\cdot,\cdot>)$ Prove that for every x in V, there exists unique y in W and z in $W^{\perp}$ such that $x=y+z$. Here $W^{\perp}$ is the orthogonal complement of W.