淡江大學 101 學年度轉學生招生考試試題

系別：數學學系三年級　　科目：高等微積分

考試日期：7 月 17 日(星期二) 第 4 節　　本試題共 8 大題，1 頁

1. (16 points) Let f be defined by

$$f(x)= x^2 \sin(\frac{1}{x}) \text{ if } x\neq 0 \text{ and } f(0)=0.$$

(a) Prove that f is differentiable at 0 and that $f'(0)=0$.

(b) Show that $f'(x)$ is not continuous at 0.

2. (8points) Find $\frac{dy}{dx}$ if $y=\int_0^x \sqrt{1+t^2}\,dt$.

3. (16 points) Let $f(x)=\sqrt{x}$, $0\leq x$.

(a) Show that $f(x+y)\leq f(x)+f(y)$ and $|f(x)-f(y)|\leq f(|x-y|)$.

(b) Show that f(x) is uniformly continuous on $[0, \infty)$.

4. (12 points) Show that $\sum_{k=1}^{\infty}\frac{1}{2^k+x^2}$ converges uniformly on the real line.

5. (12 points) If $f: R\to R$ is differentiable and $z = f(x-y)$, show that $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$.

6. (12 points) Let $f: [0, 1]\to [0, 1]$ be continuous. Prove that there is c in [0, 1] such that f(c)=c.

7. (12 points) Let f and g be real-valued functions defined on a nonempty set X satisfying Range(f) and Range(g) are bounded subset of R. Prove that

$$\sup\{f(x)+g(x): x\in X\}\leq \sup\{f(x): x\in X\}+ \sup\{g(x): x\in X\}.$$

8. (12 points) Let f(x) =sin x. Prove that $|f(x)-f(y)|\leq|x-y|$ for all x, y .