淡江大學 101 學年度轉學生招生考試試題

系別：數學學系三年級　　　　　　科目：代　　數

考試日期：7 月 17 日(星期二) 第 3 節　　　　　本試題共　10　大題，　1　　頁

Show your work　每題十分

1. Let G be a group. If $(ab)^2 = a^2 b^2$ for every a, b in G, show that G is abelian.

2. Find the remainder when 75^{22} is divided by 23.

3. Show that $Z_2 \times Z_3$ is a cyclic group.

4. Show that every group of order 37 is abelian.

5. Let G be a finite group and $800 \le |G| \le 1000$. If G has two subgroups H_1 and H_2 with

   $|H_1| = 45, |H_2| = 75$, then what is $|G|$?

6. Let H, K be subgroups of a group G. If $K \triangleleft G$, prove that $HK = KH = G$.

7. Let R be a ring, $a^2 = a$ in R, show that $(1-a)ra$ and $ar(1-a)$ are nilpotents for $r \in R$.

8. Let R be a commutative ring, an ideal $P \ne R$. Prove that P is prime if and only if R/P is an integral domain.

9. Show that there is no field of order 6.

10. If $F = \{0, 1, a, b\}$ is a field, fill in the addition and multiplication tables for F..