2-4

淡江大學100學年度轉學生招生考試試題

29-1

系別：數學學系三年級　　　　科目：線性代數

考試日期：7月19日(星期二) 第1節　　　　本試題共 8 大題，1/2 頁

1. (15 points) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$.

   (a) Find the matrix representation of T with respect to the standard basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ of $\mathbb{R}^3$.

   (b) If (a, b, c) is a vector in $\mathbb{R}^3$, what are the conditions on a, b, c that the vector is in the range of T? What is the rank of T?

   (c) What are the conditions on a, b, c that the vector (a, b, c) is in the null space of T? What is the nullity of T?

2. (10 points) Let $u = (2,1,0)$, $v = (3,0,2)$ and $w = (0,-2,3)$. Suppose that T is a linear operator on $\mathbb{R}^3$ that interchanges u and v, and maps w to $(1,0,0)$. Find the matrix representation $[T]_\mathcal{B}$ of T with respect to the standard basis $\mathcal{B} = \{(1,0,0),(0,1,0),(0,0,1)\}$.

3. (10 points) Let V be the space of $n \times n$ matrices over $\mathbb{R}$.

   (a) Show that for any $A \in V$, the set $\{I, A, A^2, \cdots, A^n\}$ is linearly dependent.

   (b) Show that A is invertible if and only if I belongs to Span$\{A, A^2, \cdots, A^n\}$.

4. (15 points) Let T be a linear operator on a finite dimensional vector space V. Suppose T is idempotent, that is $T^2 = T$. Prove that

   (a) Eigenvalues of T are either 0 or 1.

   (b) $V = \ker(T) \oplus \text{range}(T)$.

   (c) T is diagonalizable.

5. (15 points) (a) Let $T : V \longrightarrow W$ be a linear transformation from vector space V to vector space W. Show that T is nonsingular (1-1) if and only if T maps a linearly independent set of vectors in V to a linearly independent set of vectors in W.

   (b) Let $T : V \longrightarrow W$ be a linear transformation from vector space V to vector space W. Suppose $\dim V = \dim W$. Show that T is one to one if and only if T is onto.

6. (15 points) Let $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Determine whether A similar to a diagonal matrix over $\mathbb{R}$. If so, exhibit a basis for $\mathbb{R}^3$ such that A is similar to a diagonal matrix.

7. (10 points) Let A be a $n \times n$ matrix over the field $\mathbb{F}$. Let λ_1, λ_2 be two distinct eigenvalues of A and W_1, W_2 be the corresponding eigenspaces for λ_1, λ_2 respectively. Show that $W_1 \cap W_2 = \{0\}$.

本試題雙面印刷

背面尚有試題

淡江大學100學年度轉學生招生考試試題

系別：數學學系三年級　　　　科目：線 性 代 數

考試日期：7月19日(星期二) 第1節　　　　本試題共 8 大題，2/2 頁

8. (10 points) Let V be a finite dimensional vector space over a field F and dim $V \geq 2$. Let $T : V \longrightarrow V$ be a linear transformation. If there exists a vector $v \in V$ such that V is spanned by v, $T(v)$, $T^2(V), \cdots$, prove that the characteristic polynomial of T is equal to it minimal polynomial.