1. (20%) A function f with domain S is said to be uniformly continuous on S if and only if for every $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - f(y)| < \varepsilon$ whenever $|x - y| < \delta$, $x, y \in S$.

   (a) show that $f(x) = x^2$ is uniformly continuous on the interval $(0,1)$.

   (b) show that $f(x) = x^2$ is not uniformly continuous on $\mathbb{R}$.

2. (20%) A subset H of a metric space X is said to be compact if and only if every open covering of H has a finite subcover.　Show that a closed subset of a compact set is compact.

3. (20%) (a) Let $f(x) = |x|$.　Show that f is not differentiable on $[-1,1]$.

   (b) Find $\dfrac{d}{dx} \displaystyle\int_0^{2x} t \cos t \, dt$.

4. A sequence of functions $f_n : E \to \mathbb{R}$, where E is a subset of $\mathbb{R}$, is said to converges uniformly on E to a function f if and only if for every $\varepsilon > 0$ there is a $N \in \mathbb{N}$ such that

$$n \in \mathbb{N} \text{ implies } |f_n(x) - f(x)| < \varepsilon$$

for all $x \in E$.　Let $f_n(x) = \begin{cases} n^2 x & , 0 \le x < \frac{1}{n} \\ 2n - n^2 x & , \frac{1}{n} \le x < \frac{2}{n} \\ 0 & , \frac{2}{n} \le x < 1 \end{cases}$

   (a) Find $\displaystyle\lim_{n \to \infty} f_n(x)$

   (b) Find $\displaystyle\int_0^1 f_n(x) \, dx$

   (c) Find $\displaystyle\lim_{n \to \infty} \int_0^1 f_n(x) \, dx$

   (d) Show that f_n doesn't converges uniformly.

   (Hint: If $f_n \to f$ uniformly on $[0,1]$, then $\lim_{n \to \infty} \int_0^1 f_n(x) \, dx = \int_0^1 f(x) \, dx$.)

5. (20%) Let $f(x,y) = \begin{cases} xy \left(\frac{x^2 - y^2}{x^2 + y^2} \right) & , (x,y) \neq 0 \\ 0 & , (x,y) = 0 \end{cases}$

   (a) Find $f_x(0,0)$ and $f_y(0,0)$.

   (b) Find $f_{xy}(0,0)$ and $f_{yx}(0,0)$.