

# 淡江大學 100 學年度轉學生招生考試試題

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系別：數學學系三年級

科目：代 數

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考試日期：7月19日(星期二) 第3節

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Show Your work (20 points each)

1. Find the remainder when:

(a)  $25^{49}$  is divided by 11(b)  $23^{461}$  is divided by 432. Show that a group  $G$  is abelian if and only if  $x^2 = e$  for any  $x$  in  $G$ , where  $e$  is the identity of  $G$ .3. Let  $G$  be a group such that  $|G| < 200$ . Suppose  $G$  has subgroups of order 33 and 55, find the order of  $G$ .4. If  $H$  is a subset of a group  $G$ , show that  $H$  is a subgroup of  $G$  if and only if  $H$  is nonempty and  $ab^{-1} \in H$  whenever  $a, b \in H$ .5. Let  $\alpha : G_1 \rightarrow G_2$  be a group homomorphism, prove that(a)  $\alpha(G_1) \leq G_2$ (b)  $\ker \alpha \triangleleft G_1$ (c)  $\ker \alpha$  is trivial  $\Leftrightarrow \alpha$  is 1-1