淡江大學99學年度轉學生招生考試試題

系別：數學學系三年級　　　　　　　　科目：：線 性 代 數

◆ 必須寫出計算過程。

本試題雙面印刷

1. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be linear transformation defined by $T(x, y, z) = (-2x+2y-z, x-2y-3z, 5x-3y+2z)$.

   (a) (5 points) Find the matrix representation of T in the standard basis of $\mathbb{R}^3$.

   (b) (10 points) Give a basis for the kernel and range of T.

2 .Let V be the vector space of polynomials of degree at most 2. Let $\mathcal{B} = \{1, x, x^2\}$ and $\mathcal{B}' = \{x, 1+x, -x+x^2\}$. Let $T : V \longrightarrow V$ be the map defined by $T(p(x)) = p''(x) - 2p'(x)$ where $p'(x)$ denotes the derivative and $p''(x)$ denote the second derivative of $p(x)$.

   (a) (5 points) Show that T is a linear transformation.

   (b) (5 points) Find the matrix representation A of T relative to the basis $\mathcal{B}$.

   (c) (5 points) Find the change of coordinate matrix from $\mathcal{B}$ to $\mathcal{B}'$.

   (d) (10 points) Find the matrix representation A' of T relative to $\mathcal{B}'$.

3. Let $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ -2 & -2 & 1 & 1 \\ -1 & 1 & 2 & 1 \end{bmatrix}$.

   (a) (10 points) Find the eigenvalues and the corresponding eigenvectors of A.

   (b) (5 points) Find the minimal polynomial of A.

4. (10 points) Let $\{v_1, v_2, \cdots, v_n\}$ be a basis for the vector space V. Suppose that $w \in V$ is not in the subspace spanned by $\{v_1, v_2, \cdots, v_{n-1}\}$. Show that $\{v_1, v_2, \cdots, v_{n-1}, w\}$ is also a basis of V.

4. For any $n \times n$ matrices A, B,

   (a) (10 points) Matrix AB and matrix BA have the same eigenvalues.

   (b) (5 points) Show that $AB - BA \neq I_n$, where I_n denote the $n \times n$ identity matrix.

背面尚有試題

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5. (20 points) Let $T : V \longrightarrow V$ be linear transformation of the vector space V. Prove or disprove the following statements.

(a) If $\{v_1, \cdots, v_n\} \subset V$ is linearly independent, then $\{T(v_1), \cdots, T(v_n)\}$ is linearly independent.

(b) If $\lambda_1, \cdots, \lambda_k$ are k distinct eigenvalues of T and $v_1, \cdots, v_k$ are the eigenvectors corresponding to $\lambda_1, \cdots, \lambda_k$ respectively, then $v_1, \cdots, v_k$ are linearly independent.