淡江大學 99 學年度轉學生招生考試試題

系別：數學學系三年級　　　　　　　　科目：：高 等 微 積 分

本試題共 八 大題，一 頁

1. (12 points) Show that $f(x) = \sin x$ is uniformly continuous on $(-\infty, \infty)$.

2. (12 points) Let $f : (0,2) \to \mathbb{R}$ be continuous and $f'(x)$ exist for all x in $(0,1) \cup (1,2)$. If $\lim_{x\to 1} f'(x) = L$, show that $f'(1)$ exists and $f'(1) = L$.

3. (12 points) Let $f : [0,1] \to \mathbb{R}$ be continuous and $f(x) \geq 0$ for all $x \in [0,1]$. If $\int_0^1 f(x)dx = 0$, show that $f(x) = 0$ for all x.

4. (12 points) Let $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$ for all $n \in \mathbb{N}$, show that $\lim_{n\to\infty} a_n$ exists and evaluate it.

5. (16 points)

   (a) (8 points) Show that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all real x.

   (b) (8 points) Estimate $\int_0^1 e^{-x^2/2}dx$ to three decimal place.

6. (12 points) Evaluate $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}$.

7. (12 points) Let

$$u(x,y,z) = x + xyz$$
$$v(x,y,z) = y + xy$$
$$w(x,y,z) = z + 2x + 3z^2$$

   Show that x, y, z can be solved in terms of u, v, w near $(0,0,0)$ and find $\partial y/\partial v$ and $\partial z/\partial u$ at $(0,0,0)$.

8. (12 points) Let

$$f(x,y) = \begin{cases} (x^2+y^2)\sin\frac{1}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

   Show that f is differentiable at $(0,0)$ while $\partial f/\partial x$ is not continuous at $(0,0)$.