淡江大學99學年度轉學生招生考試試題

系別：數學學系數學組三年級　　　　科目：：代　　　　數

Show Your work.

1. Determine which of the following groups is cyclic: (20%)

   (a) Z_{30} (b) $Z_4 \times Z_6$ (c) Z_{18}^* (d) Z_7^*.

2. Show that every cyclic group is abelian. (10%)

3. Show that a finite integral domain is a field. (10%)

4. Show that $Z(\sqrt{2}) = \{m + n\sqrt{2} \mid m, n \in Z\}$ is a subring of C and find 14 units. (20%)

5. Prove or disprove that a UFD is a PID. (10%)

6. Let R be a commutative ring and M an ideal of R, prove that M is maximal if and if R/M is a field.

   (10%)

7. Prove or disprove that every prime is irreducible in an integral domain. (10%)

8. Prove that a group G is abelian if $(ab)^2 a^2 b^2$ for every a,b in G. (10%)