系別：數學學系三年級　　　　　　科目：線　性　代　數

准帶項目請打「V」
計　算　機

本試題共　6　大題，　　　頁

Transfer Exam. for Linear Algebra 2009/7/15

1. Solve
$$-2x + 3y + 3z = -9$$
$$3x - 4y + z = 5$$
$$-5x + 7y + 2z = -14 \quad \text{by Gaussian elimination.} \quad (10\%)$$

2. Solve
$$x + 2y - z = 2$$
$$2x + 5y - 3z = 1$$
$$x + 4y - 3z = 3 \quad \text{by Gaussian elimination.} \quad (10\%)$$

3. (a) Show that every square matrix A can be written as A = S + W, where S is symmetric and W is skew-symmetric.

   (b) Write $A = \begin{bmatrix} 2 & -3 & 5 \\ 1 & 7 & -4 \\ -11 & 2 & 6 \end{bmatrix} = $ S + W, where S is symmetric and W is

   skew-symmetric.　　　　　　　　　　　　　(20%)

4. Let $A = \begin{bmatrix} 2 & 7 & 1 \\ 1 & 4 & -1 \\ 1 & 1 & 0 \end{bmatrix}$,　　　　　　　　　(20%)

   (a) Find $|A|$ (determinant of A).

   (b) Find A^{-1}.

5. Let T: $R^4 \to R^3$ be a linear transformation by $T(\vec{X}) = A\vec{X}$, where A

   $= \begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & 1 & 4 & 3 \\ 5 & 6 & 1 & 5 \end{bmatrix}$. Find bases for kerT and imT.　　(20%)

6. Diagonalize A, if possible. (20%)

   (1) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & -3 \\ 1 & -1 & 0 \end{bmatrix}$　(2) $A = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$