淡江大學98學年度轉學生招生考試試題

系別：數學學系三年級　　　　　科目：高 等 微 積 分

准帶項目請打「V」	
	計 算 機

本試題共 6 大題，　　頁

1. (20%) A function f with domain S is said to be uniformly continuous on S if and only if for every $\varepsilon > 0$ there is $\delta > 0$ such that $|f(x) - f(y)| < \varepsilon$ whenever $|x - y| < \delta$, $x, y \in S$.

   (a) Show that $f(x) = \frac{1}{x}$ is not uniformly continuous on $[0,1]$.

   (b) Show that $f(x) = \frac{1}{x}$ is uniformly continuous on $[1,\infty)$.

2. (20%) Let $f(x,y) = \begin{cases} 0 & \text{if } (x,y) = (0,0) \\ \dfrac{xy^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \end{cases}$

   (a) Find the first partial derivative of f with respect to x at (0,0), namely, $D_1 f(0,0)$.

   (b) Find the first partial derivative of f with respect y at (0,0), namely, $D_2 f(0,0)$.

   (c) Find the directional derivative of f at (0,0) in the direction of (u_1, u_2) with $u_1^2 + u_2^2 = 1$.

   (d) Is f differentiable at (0,0)? Explain your answer.

3. (15%) Let $\{y_n\}$ be the sequence in R defined inductively by

   $y_1 = 1$, $\quad y_{n+1} = (2y_n + 3)/4$.

   (a) Show that $0 < y_n \le 2$ by induction.

   (b) Show that $y_n < y_{n+1}$ by induction.

   (c) Find $\lim_{n\to\infty} y_n$.

4. (15%) Let $F: R^5 \to R^2$ be defined by

   $F(x,y,u,v,w) = (2e^x + yu - 4v + 3, y\cos x - 6x + 2u - w)$.

   If $(x,y) = (0,1)$ and $(u,v,w) = (3,2,7)$, then $F(x,y,u,v,w) = 0$. Show that we can solve $F(x,y,u,v,w) = (0,0)$ for (x, y) in terms of (u, v, w) near $(3,2,7)$.

5. (10%) (a) Find $\frac{dy}{dx}\int_0^{2x} (t^2 \sin t)\,dt$.

   (b) Find $\frac{dy}{dx}\int_x^{2x} (t^2 \sin t)\,dt$.

6. (20%) We say that a sequence of function $\{f_n\}$, $n = 1, 2, 3, \ldots$ converges uniformly on $E \subseteq R$ to a function f if for every $\varepsilon > 0$ there is an integer N such that $n \ge N$ implies $|f_n(x) - f(x)| < \varepsilon$ for all $x \in E$.

   (a) Let $f_n(x) = \frac{1}{n}\sin(nx+n)$ on R. Find the limit function f.

   (b) Show that $f_n(x) = \frac{1}{n}\sin(nx+n)$ converges to the limit function f uniformly on R.

   (c) Find an example of a sequence of functions $\{f_n\}$ defined on R that are not uniformly convergent.