淡江大學 97 學年度轉學生招生考試試題

系別：數學學系三年級　　　　科目：線 性 代 數

可否使用計算機			
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本試題共 七 大題， 一 頁

1. Let $W=\{(a_1,a_2,a_3)\in R^3 \mid a_1+2a_2+a_3=0, a_1+a_2+3a_3=0\}$. Find the basis (基底) for W and the dimension (維度) of W. (10%)

2. Let $A=\begin{bmatrix}0&2&4\\2&4&2\\3&3&1\end{bmatrix}$. Calculate the determinant (行列式) of A and find its inverse (反矩陣). (10%)

3. Let $A=\begin{bmatrix}1&3&3\\-3&-5&-3\\3&3&1\end{bmatrix}$.

   (a) Find the characteristic polynomial (特徵多項式), the eigenvalues (特徵值), and the corresponding eigenvectors (特徵向量) of A. (10%)

   (b) Find an invertible matrix P and a diagonal matrix (對角矩陣) D such that $D=P^{-1}AP$. (10%)

4. Let $P_2(R)$ be the set of real polynomials with degree ≤ 2 and $M_{2\times 2}(R)$ be the set of all 2×2 real matrices. Define the linear transformation $T:P_2(R)\to M_{2\times 2}(R)$ by $T(p(x))=\begin{bmatrix}p(0)&0\\0&p(1)-p(3)\end{bmatrix}$.

   (a) Find the range of T. (10%)

   (b) Find the null space of T. (10%)

   (c) Show that $\dim(P_2(R))=rank(T)+nullity(T)$. (3%)

5. $\beta=\{(1,-1,0,0)^T,(0,-1,0,1)^T,(0,-1,1,0^T)\}$ is a basis for the plane $W: x_1+x_2+x_3+x_4=0$ in R^4. Consider the standard inner product (標準內積) on R^4.

   (a) Apply the Gram-Schmidt process on β to find an orthonormal (單範正交) basis for the plane W. (10%)

   (b) Find the orthogonal projection (正交投影) of $(1,1,0,1)^T$ on W. (10%)

6. Let $T:R^n\to R^m$ be a linear 1-1 transformation. Suppose that $\{u_1,\ldots,u_k\}$ is a linearly independent (線性獨立) set of vectors in R^n. Show that $\{Tu_1,\ldots,Tu_k\}$ is linearly independent. (10%)

7. A, B and C are $n\times n$ real matrices and $A=BC$. Show that A is invertible(可逆) if and only if B and C are invertible. (7%)