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# 淡江大學97學年度轉學生招生考試試題

系別：數學學系三年級　　　　科目：高等微積分

| 可否使用計算機 | | | |
|---|---|---|---|
| 可 | | 否 | ✓ |

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1. Show that f(x) = $\frac{1}{x}$ is not uniformly continuous on (0, 1). (10 points).

2. Let g be defined by g(x, y)= $\frac{xy(y^2-x^2)}{x^2+y^2}$ if (x, y)≠(0,0) and g(0,0)= 0. Find $\frac{\partial^2 g}{\partial x\partial y}(0,0)$ and $\frac{\partial^2 g}{\partial y\partial x}(0,0)$. (10 points)

3. If f : R→ R is differentiable and z = f(x-y), show that $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$. (10 points)

4. Let K be a compact subset of R and let f be a real –valued function on K. Prove that if f is continuous on K, then f (K) is compact. (10 points)

5. Let f(x, y)= (xy , $x^2+y^2$).

   (1). Show that g= $f^{-1}$ exists and is differentiable in some nonempty open set containing (2,5). (10 points)

   (2). Find Dg(2,5)(the total derivative of g at(2,5)). (10 points)

6. Let f be continuous on [a, b]. Show that $\int_a^c f(x)\,dx = 0$ for all c in [a, b] if and only if f(x) = 0 for all x in [a, b]. (15 points)

7. Let f : [0, 1]→ [0, 1] be continuous. Prove that there is c ∈ [0, 1] such that f(c)=c. (10 points)

8. Find $\frac{dy}{dx}$ if y= $\int_0^{2x}\sqrt{1+t^2}\,dt$. (5 points)

9. Let $\{f_n\}$ be a sequence of real-valued functions on [0, 1] and $f_n$ converges uniformly to a function f. Prove that if each $f_n$ is continuous on [0, 1], then f is continuous on [0, 1]. (10 points)