淡江大學97學年度轉學生招生考試試題

系別：數學學系數學組三年級　　　　科目：代　　　數

可否使用計算機			
可		否	X

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1. (24 points)

Let $G = < a >$ be a cyclic group of order n.

(a) Show that if H is a subgroup of G, then $H = < a^r >$ for some $r \in \mathbb{Z}$.

(b) Show that for each $d \mid n$, there is a subgroup of order d.

(c) If $\gcd(r, n) = 1$, show that $< a^r > = G$.

(d) $m \in \mathbb{N}$. Find the order of the subgroup $< a^m >$.

2. (16 points)

(a) Show that the set of integers $\mathbb{Z}$ is a principal ideal domain.

(b) Show that non-zero prime ideals in $\mathbb{Z}$ are of the form (p) where p is a prime number in $\mathbb{Z}$.

4. (20 points)

Let R be a commutative ring with identity.

(a) Let $\mathcal{M} \subset R$ be an ideal in R. Show that $\mathcal{M}$ is a maximal ideal in R if and only if $R/\mathcal{M}$ is a field.

(b) Suppose $R = \mathbb{R}[x]$ is the polynomial ring with real coefficients. Show that $(x^2 + 1)$ is a maximal ideal in R and $\mathbb{R}[x]/(x^2 + 1) \cong \mathbb{C}$.

5. (10 points)

Let G be a group. Show that G is abelian if and only if the map $\varphi : G \longrightarrow G$ given by $\varphi(a) = a^2$ is a group homomorphism.

6. (10 points)

Show that groups of order 4 are abelian.

7. (20 points)

Let H, K be subgroup of the group G.

(a) Give an example to show that $HK = \{hk \mid h \in H,\ k \in K\}$ may not be a subgroup.

(b) Show that if H is a normal subgroup of G, then HK is a subgroup of G.