淡江大學96學年度轉學生招生考試試題

系別：數學學系三年級　　　　科目：線性代數

可否使用計算機			
可	✓	否	

本試題共 2 頁

1. Let $A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$

(a) (10 points) Find the characteristic polynomial (特徵多項式), the real eigenvalues (特徵值), and the corresponding eigenvectors (特徵向量) of A.

(b) (10 points) Find an invertible matrix P and a diagonal matrix (對角矩陣) D such that $D = P^{-1}AP$

2. (a) (10 points) Find all values of a for which the inverse of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}$ exists. What is A^{-1}?

(b) (10 points) If $A^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & -2 \\ 1 & 1 & -1 \end{bmatrix}$

find the solution of the linear system

$$(AB)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

3. (a) (10 points) Find bases for the row and column spaces of $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$

(b) (5 points) Find a basis of the nullspace of $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$

本試題雙面印製

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4. (10 points) Suppose $S=\{v_1, v_2, \cdots, v_n\}$ is an orthogonal set of nonzero vectors in an inner product space. Show that S is linearly independent.

5. (10 points) Let $L: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Let $S=\{v_1, v_2, v_3\}$, $T=\{w_1, w_2\}$ where $v_1=\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_2=\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $v_3=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $w_1=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $w_2=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Find the matrix of L with respect to S and T.

6. (10 points) Suppose u, v are vectors ... a real inner product space, $\langle u, v\rangle$ is inner product of vectors u, v, $\|u\| = \langle u, u\rangle^{1/2}$. show that
$$|\langle u, v\rangle| \le \|u\|\,\|v\|.$$

7. (a) (5 points) If $A^TA = A$ show that A is symmetric and $A = A^2$

(b) (5 points) If $A = A^2$, and $A \neq I$, show that A^{-1} don't exist.

(c) (5 points) If $A = [a_{ij}]_{n\times n}$ then $Tr(A) = \sum_{i=1}^{n} a_{ii}$. Show that $Tr(AB) = Tr(BA)$.