

系別：數學學系三年級

科目：高等微積分

| 可否使用計算機 | | | |
|---------|--|---|---|
| 可 | | 否 | ✓ |

本試題共 / 頁

1. Find the limit if it exists? Otherwise prove the limit does not exist.
- (a) $\lim_{(x,y) \rightarrow (0,0)} \frac{\ln(x^2+y^2)}{x^2+y^2}$ (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x \cdot y^3}{x^2+y^6}$ (1090)
2. (a) $\frac{d}{dx} \int_x^{x^2} [t^2 \sin t] dt = ?$ (b) $\int_0^1 \left[\frac{d}{dx} \left\{ \frac{e^{\sin^2 x} (x^3 + 3x^2 + 5x)}{(x^2+1)^{\frac{7}{3}}} \right\} \right] dx = ?$ (1090)
3. If $s_1 = \sqrt{2}$, $s_{n+1} = \sqrt{2 + s_n}$ for $n=1, 2, \dots$. Prove that $\{s_n\}$ converges and find $\lim_{n \rightarrow \infty} s_n$. (1090)
4. Suppose $|f(x) - f(y)| \leq |x - y|^{\frac{3}{2}}$. Prove or disprove that f is constant. (1090)
5. Let $f(x, y) = \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$ (2090)
- (a) prove or disprove that f is continuous at $(0, 0)$
 (b) prove or disprove that f is differentiable at $(0, 0)$
6. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, where $f = (f_1, f_2)$ with $f_1(x, y) = e^x \cos y$, $f_2(x, y) = e^x \sin y$. Put $a = (0, \frac{\pi}{3})$ and $b = f(a)$. (2090)
- (a) explain that there exists a neighborhood of b such that $g = f^{-1}$ exists on that neighborhood.
 (b) Compute $g'(b)$.
7. State the implicit function theorem (1090)
8. Give an example of a sequence of functions $\{f_n\}$ defined on $[0, 1]$ that are continuous but $f = \lim_{n \rightarrow \infty} f_n$ is not continuous. (1090)