淡江大學95學年度轉學生招生考試試題

33

系別：數學學系三年級　　　　科目：線性代數

33-1

准帶項目請打「V」	
V	簡單型計算機

本試題雙面印製

1. Consider the following linear system

$$\begin{cases} 3x + 4y + 3z = 0 \\ \quad\; - y - z = 1 \\ x - 2y - 2z = 0 \end{cases}$$

   (a) (5 points) Write down A and b such that the system has the form $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = b$.

   (b) (10 points) Calculate the determinant(行列式) of A, and show that A is invertible(可逆).

   (c) (10 points) Find A^{-1}.

   (d) (5 points) Solve $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ by A^{-1}.

2. (10 points) Suppose that $\{v_1, \cdots, v_n\}$ is a linearly independent(線性獨立) set of vectors in $\mathbb{R}^n$. Let A be a $n \times n$ nonsingular matrix. Show that $\{Av_1, \cdots, Av_n\}$ is linearly independent.

3. Let $A = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{bmatrix}$.

   (a) (10 points) Find the characteristic polynomial(特徵多項式), the real eigenvalues(特徵值), and the corresponding eigenvectors(特徵向量) of A.

   (b) (10 points) Find an invertible matrix C and a diagonal matrix(對角矩陣) D such that $D = C^{-1}AC$.

4. (10 points) Let L_1 and L_2 be linear transformation from a vector space V into a vector space W. Let $\{b_1, \cdots, b_n\}$ be a basis for V. Show that if $L_1(b_i) = L_2(b_i)$ for all i, then $L_1(v) = L_2(v)$ for all $v \in V$.

◀注意背面尚有試題▶

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5. (10 points) Let P_2 be the set of polynomials with degree less than or equal to 2. Find a basis for the subspace of P_2 consisting of all vectors of the form $ax^2 + bx + 3a - 4b$.

6. (10 points) Use the Gram-Schmidt process to transform the basis $\{(1,1,1),(0,1,1),(1,2,3)\}$ for $\mathbb{R}^3$ into an orthonormal basis for $\mathbb{R}^3$.

7. (10 points) Let $L : P_2 \to P_2$ be the linear transformation defined by

$$L(ax^2 + bx + c) = (a+b)x + b + 2c.$$

(a) (5 points) Is $-2x^2 + 2x - 4$ in kernal of L?

(b) (5 points) Is $3x^2 + 1$ in range of L?